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# ANTENNA IMPEDANCE IN THE IONOSHERE

by

Alvin M. Despain

Scientific Report No. 3

Contract No. AF 19(628)-4995
Project No. 7663
Task No. 766303
Contract Monitor: James C. Ulwick

13 May 1966

This research was sponsored, in part, by the Defense Atomic Support Agency, Washington, D.C. under Web No. 07.007, Project 5710.



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University of Utah

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Submitted by Obed C. Haycock
Obed C. Haycock, Director

## LIST OF CONTRIBUTORS

Obed C. Haycock, Director

Kay D. Baker, Assistant Director

## RELATED CONTRACTS

AF 19(628)-447

AF 19(628)-229

## RELATED PUBLICATIONS

"A Plasma Resonance and Electron Temperature Experiment," Scientific Report No. 3, Contract No. AF 19(628) 447, AFCRL No. 64-809, August 1964.

"The Theory of Cylindrical Antenna Impedance as a Function of Plasma Frequency and Temperature," Scientific Report No. 1, Contract No. AF 19 (628)-4995, March 1965.

"Direct *In Situ* Measurements of Auroral Parameters," Scientific Report No. 4, Contract No. AF 19(628)-4995, May 1966.

"RF Probe Techniques," Final Report, Contract No. AF 19(628)-447, 1965.

"Measurement of Missile Trail Ionization by the Drop-Out Probe Technique," Final Report under Contract No. AF 19(628)-229, 21 June 1965.

"Experiences with Antenna Impedance Measurements in the Ionosphere," paper presented at the 1965 Fall URSI Meeting, Dartmouth College, Hanover, New Hampshire, 5 October 1965.

"Ionospheric Electron Temperatures from Antenna Impedance Measurements," paper presented at the 1966 Spring URSI Meeting, National Academy of Sciences, Washington D.C., 19 April 1966.

"Experimental and Theoretical Studies of Antenna Impedance in the D and E Regions," paper presented at the Second Conference on Direct Aeronomic Measurements in the Lower Ionosphere, University of Illinois, Urbana, Illinois, September 28, 1965.

"Simultaneous Comparison of RF Probe Techniques for Determination of Ionospheric Electron Density," *J. Geophys. Res.*, *71*, 935-949, February 1966.

## ACKNOWLEDGEMENT

Sincere appreciation is expressed to the many individuals who have been of assistance in the pursuit of the research reported here and in the preparation of this paper.

The author is deeply indepted to all the staff of the Upper Air Research Laboratories, especially to Professor Obed C. Haycock and Mr. Kay D. Baker for guidance and support. Mr. Grant Duffin, Mr. Kent Olsen, Mr. Michael Taylor and Mr. Wayne Wride have all provided valuable aid in data reduction while Mr. Glenn Allred, Mrs. Lonnie Dalrymple, Mr. Robert Fowler, Mrs. Diane Hansen, Mrs. Susan Lambert and Mr. Andre Schwager have materially assisted in the preparation of this report.

Gratitude is also due to Mr. James C. Ulwick and Dr. Wolfgang Pfister of the Air Force Cambridge Research Laboratories for providing scientific consultation and support.

The University of Utah Computer Center gave valuable assistance in preparation and execution of several computer studies.

# ABSTRACT

The impedance of an antenna probe surrounded by ionospheric plasma is examined. A review of several physical plasma parameters, research techniques, some past experimental results, and various theories is presented. A need is shown for further theoretical sophistication. The theory develops the impedance of an electrically short, cylindrical antenna probe immersed in a warm, lossy, compressible, magneto-ionic, electron fluid. The results are examined and compared to previous work and are applied to several ionospheric experiments. Experimental results as interpreted by the theory are examined and the usefulness of the theory is thus demonstrated. In particular, the theory is shown to be of value in determining electron density, temperature, and collision frequency from RF probe measurements.

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## LIST OF SYMBOLS AND USEFUL RELATIONS

[A] = a tensor operator

$a^2$ = $K'/K_o$

$\alpha^2$ = $(\psi^2 - 1)\omega_N^2/\mu^2$

$\alpha'^2$ = $-\alpha^2$

$\vec{B}$ = magnetic field vector

$\beta$ = $\omega/c$

$\beta(x)$ = a special function

$\beta_n$ = $\mu^2/UN\omega$

$\beta_p$ = $e/Um\omega$

$\beta_r$ = a special operator

$\beta_\theta$ = a special operator

C = capacitance

C' = a general purpose constant

$C_N$ = model atmosphere constant

$C_s$ = sheath capacitance

$C_o$ = free space capacitance

$C_1$ = arbitrary constant

$C_2$ = $-en_2/\varepsilon_o\alpha^2$

c = velocity of light ($2.997930 \times 10^8$ m/sec)

$\vec{D}$ = electric flux vector

d = differential operator

$d\vec{S}$ = a differential surface element vector

dV = a differential volume element

[$\Delta$] = a tensor operator

$\nabla$ = a differential space operator

$\partial$ = a partial differential operator

$\vec{E}$ = electric field vector

e = electronic charge ($1.60206 \times 10^{-19}$ coulombs)

$\varepsilon$ = dielectric permittivity

[$\varepsilon$] = dielectric permitivity tensor

$\varepsilon_o$ = permittivity of free space ($8.85434 \times 10^{-12}$ farads/m)

F = $\text{Sin}^2\alpha + a^2\text{Cos}^2\alpha$

$F_c$ = effective force due to collisions

$F_L$ = Lorentz Force

$F_P$ = force due to pressure

f = operating, exciting or applied frequency

f' = a given frequency

$f_H$ = gyrofrequency (aprox. 1.4 MHz in ionosphere)

$f_N$ = plasma frequency

$f_p$ = parallel resonance frequency

$f_s$ = series resonance frequency

$\gamma$ = angle between antenna & earth's magnetic field

$\gamma'$ = ratio of specific heats

$\vec{H}$ = magnetic field strength vector

$H(x)$ = Hankel function of the first kind

$H^{(2)}$ = Hankel function of the second kind

$[I]$ = unit tensor

$I_a$ = current at antenna terminals

$J_D$ = displacement current density

$j$ = $\sqrt{-1}$

$K$ = Boltzmann's constant ($1.38044 \times 10^{-23}$ Joules/°K)

$K$ = $1 - XU/(U^2-Y^2)$

$K(x)$ = modified Bessel function of second kind

$K_o$ = $1 - X/U$

$L$ = antenna length

$\lambda$ = wavelength

$\lambda_o$ = Debye length

$M$ = molecular mass

$m$ = electronic mass ($9.1083 \times 10^{-31}$ kg)

$\mu$ = velocity associated with longitudinal waves

$\mu^2$ = $\gamma' KT/m$

$\mu_o$ = permeability of free space ($4\pi \times 10^{-7}$ Henry/m)

$N$ = electron density

$N_t$ = total particle density

$n$ = variation in electron density

$n_o$ = initial electron density variation

$\nu$ = electron collision frequency

$\omega$ = radian operating, exciting or applied frequency

$\omega_H$ = radian gyrofrequency

$\omega_N$ = radian plasma frequency

$\vec{P}$ = polarization vector

$P_r$ = radiated power

$p$ = pressure

$\pi$ = 3.14159265

$\Phi$ = electric potential

$\phi(x)$ = $H_o(x)/xH_1(x)$

$Q$ = total electric charge

$Q_1$ = radiated power per unit length

$R$ = antenna radius

$R_a$ = antenna resistance

$R_L$ = $2\pi\epsilon_o LR_s\sqrt{1-x}/x\beta^2\mu C_o^2$

$R_r$ = radiation resistance

$R_s$ = sheath radius

$r$ = radius coordinate of cylindrical coordinate system

$\rho$ = mass density

$S$ = sheath scaling constant

$S(x)$ = a differential operator

$[\sigma]$ = a tensor variable

$T$ = electron temperature

$t$ = time

$t_{ij}$ = cross components

$\theta$ = angle coordinate of cylindrical coordinate system

$U$ = $1 - jZ$

$[T]$ = a tensor variable

$V$ = $\mu^2/\omega_N^2$

$V_a$ = antenna voltage

$v$ = velocity

$\vec{v}$ = velocity vector

$\bar{v}$ = mean electron velocity

$\bar{v}_m$ = mean molecular velocity

$v_\theta$ = component of $\vec{v}$ in $\theta$ direction

$v_r$ = component of $\vec{v}$ in r direction

$v_z$ = component of $\vec{v}$ in z direction

$W$ = $Y/U$

$W_\theta$ = $W\zeta_\theta$

$W_r$ = $W\zeta_r$

$W_z$ = $W\zeta_z$

$X$ = $\omega_N^2/\omega^2$

$x$ = a general variable

$\psi^2$ = $(U^2-Y^2)/UX$

$Y$ = $\omega_H/\omega$

$Y_x$ = $Y\zeta_x$

$Y_y$ = $Y\zeta_y$

$Y_z$ = $Y\zeta_z$

$Z$ = $\nu/\omega$

$Z_a$ = antenna impedance

$Z_{ar}$ = antenna impedance due to radiation resistance

$Z_{at}$ = total antenna impedance

$z$ = height coordinate of cylindrical coordinate system

$\hat{\zeta}$ = unit vector in direction of magnetic field

$\zeta_r$ = component of $\hat{\zeta}$ in r direction

$\zeta_\theta$ = component of $\hat{\zeta}$ in $\theta$ direction

$\zeta_x$ = component of $\hat{\zeta}$ in x direction

$\zeta_y$ = component of $\hat{\zeta}$ in y direction

$\zeta_z$ = component of $\hat{\zeta}$ in z direction

# Chapter I

## INTRODUCTION

It is the purpose of this report to examine the interaction between an antenna probe and the ionospheric plasma surrounding it. This introductory chapter discusses plasma and ionospheric parameters and several research techniques. Chapter II considers past theoretical work that relates antenna probe impedance to the plasma parameters. The need for further effort is then demonstrated. A theory that relates the impedance of an electrically short, cylindrical, antenna probe to the magnetic field and plasma parameters of a warm, lossy, compressible electron fluid is developed in Chapter III. The theory is explored as a function of its parameters and is compared to other accepted results in Chapter IV. Chapter V describes the application of the work to various experiments. In particular, the theory is useful in interpreting measurements of electron density, electron temperature, and electron collision frequency. The results of the analysis then demonstrate the usefulness of the theory and its broad application in ionospheric research.

It should be noted that rationalized MKS units are used throughout.

### The Plasma State

A gaseous plasma is an electrically neutral system of particles containing primarily free electrons and positive ions, but sometimes negative ions and neutral molecules as well. It has been estimated that perhaps more than 99.9 percent of the universe exists in such an ionized form [*Bachynski*, 1961]. While approximately $10^{-2}$ electron volts of energy

per particle are required to change from the solid to liquid or from the liquid to gaseous state, one to ten electron volts of energy per particle are required to obtain the plasma state. Coulomb forces are dominant in plasmas as they are much more effective than gravitational or short-range nuclear effects. Thus, the ionized particles are endowed with considerable energy and exhibit strong electrical interactions that determine the primary character of the plasma.

A system of ionized particles is considered to be a plasma only if collective effects dominate any single particle effects. Therefore, the dimensions of a plasma must be at least as large as the range of interaction of single particles. This range is called the Debye length $\lambda_D$ [*Brown et al.*, 1963] and can be expressed by the equation

$$\lambda_D = \left[\frac{\varepsilon_o KT}{Ne^2}\right]^{1/2} \tag{1}$$

Where

$K$ = Boltmann's Constant

$T$ = kinetic temperature (electrons)

$N$ = ionization density (electrons)

$e$ = electronic charge

$\varepsilon_o$ = permittivity of free space

The Debye length is a very useful parameter and is often used as a length scaling constant in normalized equations. The Debye length can also be thought of as the maximum distance over which some charge separation can occur. If any macroscopic region (large dimensions compared to $\lambda_D$), free of external influence, should have a net charge, then the charge would

create an electric field. This field would then cause the free ionized particles to redistribute into a minimum energy condition of zero field and thus zero net charge density would again result. It is not surprising then to find that the characteristic shielding distance $\lambda_D$ is directly dependent upon the energy of the electrons (electron temperature) and is inversely related to the average number of electrons that shield any discontinuities.

From the above discussion, it is apparent that disturbed plasma tends to return to equilibrium. Because the particles also have mass, the system exhibits properties characteristic of second order systems, including a natural resonant frequency, known as the plasma frequency, $f_N$. This quantity, the frequency of the "quivers" of the "plasma jelly," can be directly related to the electron density N and the physical constants electronic charge e, electronic mass m, and permittivity of free space $\varepsilon_o$ by the equation

$$\omega_N = 2\pi f_N = \left[\frac{Ne^2}{m\varepsilon_o}\right]^{1/2} \tag{2}$$

[*Brown et al.*, 1963]. The only variable in this equation is N, the electron density. Since the feature that distinguishes a plasma from other types of matter is its ionization, it is natural to expect that the electron density is the major parameter that describes the behavior of a plasma.

At this point, perhaps it is only natural to ask, "Why are the variables, electron density and temperature, the only ones specified in $\lambda_D$ and $\omega_N$? Shouldn't the positive ion density and temperature also be included?" Of course in the strict sense, the answer is that $\lambda_D$ and $\omega_N$

are defined in this manner, but it is true that shielding and natural oscillation in plasmas are indeed affected by other types of particles. However, the electrons are much less massive than the other particles (by about four orders of magnitude) and since they have the same charge, they respond to electric and magnetic fields to a much larger degree. Then too, approximate thermal equilibrium usually exists in a plasma; therefore, electrons have the same energy per particle as the other heavier particles. Thus the average electron velocity is at least two orders of magnitude larger than the ion velocity. This is easily seen by equating the mean kinetic energies as follows

$$1/2\, m\bar{v}^2 = 1/2\, M\bar{v}_m^2 \tag{3}$$

$$\bar{v} = \sqrt{\frac{M}{m}}\, \bar{v}_m \tag{4}$$

$$\bar{v} \quad 10^2\, \bar{v}_m \tag{5}$$

where

$\bar{v}$ = mean electron velocity

$\bar{v}_m$ = mean molecular velocity

$M$ = molecular mass

$m$ = electronic mass

Thus the electrons can be visualized as mobile, rapidly moving particles among relatively fixed positive ions and neutral molecules. For most of our work, it will be possible to consider a plasma as compressible electron gas, diffused among fixed ions and molecules whose only effect is to maintain average charge neutrality and to possibly thermalize the rapidly moving electrons through collisions. This will require that any exciting

frequency $\omega$ be at least as large as $10^{-1}\ \omega_N$ to $10^{-2}\ \omega_N$. Otherwise ion and molecular motions must also be considered.

A useful measure of the collisions of electrons with other paricles is the effective electron collision frequency $\nu$. It is defined as the average number of times a second that an electron suffers a collision that effectively changes its direction of motion [*Cambel*, 1963]. *Pfister* [1965] considers several definitions of electron collision frequency and relates these to the mean collision frequency as derived from the average electron velocity.

If the exciting frequency $\omega$ is considerably larger than $\nu$, then essentially no collisions occur during one cycle of excitation. Thus only one degree of freedom is possible for the electrons, that is, the direction of the original excitation. Thus, the ratio of specific heats for our compressible electron gas must be taken as 3 [*Spitzer*, 1962].

It was shown earlier that steady state charge separation does not generally exist in a plasma However, one important exception occurs at any sharp boundary where the ionization density changes markedly over a few Debye lengths. At such a surface, in the absence of electromagnetic fields, the electron flux impinging upon the boundary will greatly exceed the positive ion flux because of the much larger average electron velocity. Thus in the steady state, electric fields are set up by the accumulation of electrons on the surface, such that the ion and electron currents are equal. There results, therefore, a depletion of electrons near the boundary and a sheath (free of electrons) is said to exist between the surface and the plasma. The thickness of this sheath is on the order of a Debye length and can be modified, even dispelled, by the external application of electric potentials to the surface.

This generally draws current from the plasma and thus upsets, to some degree, the natural plasma conditions. Nevertheless, it will sometimes be assumed that sheaths around antenna probes can be collapsed so that sheath effects can be ignored.

An external magnetic field greatly modifies the behavior of a plasma. The general effect is to "stiffen" the plasma in directions normal to the direction of the field. Magnetic fields are thus responsible for the anisotropic character of such plasmas. It is possible to consider the field as a parameter of the medium, however, a related quantity, the gyrofrequency $f_H$, will often be more useful. The relation is

$$\omega_H = 2\pi f_H = \frac{eB}{m} \tag{6}$$

where

$B$ = magnetic flux density.

In summary, our physical model of the plasma state may be visualized as a warm, compressible gas of electrons, diffused among fixed ions and molecules so that macroscopic charge neutrality is maintained. The properties of the medium can be expressed by specifying the plasma parameters, that is, by stating plasma frequency $f_N$, gyrofrequency $f_H$, electron collisional frequency $\nu$, electron temperature T and the ratio of specific heats of the electron gas.

## The Ionosphere

The ionosphere is a particularly significant system which is composed of dilute, weakly ionized plasma in the earth's magnetic field. It is produced above the earth, primarily by the ionizing radiation from

the sun incident upon the earth's upper atmosphere. The structure of the ionosphere is thus strongly dependent upon the local sun angle and the level of solar activity [*Johnson*, 1961]. It is customary to identify several distinctive levels of ionization as the D, E and F layers as shown in Figure 1.

The parameters of the ionospheric plasma are illustrated in Figures 1, 2, 3 and 4. Figure 1 shows electron density and plasma frequency as a function of altitude. Figure 2 is a similar plot of electron temperature, while the expected range of Debye length is shown in Figure 3. The expected electron collision frequency is given in Figure 4. The gyrofrequency in the ionosphere is completely determined by the strength of the earth's magnetic field. This does not change radically as a function of height in this region, and the value 1.4 MHz is a representative value.

Since the time (1902) Heavyside and Kennelly [*Mitra*, 1952, p. 177] first postulated the ionosphere to explain transatlantic radio communication, measurements of the properties of the ionosphere have been of great interest. Not only are the ionospheric parameters of interest for communication studies, but they are also important to studies of other geophysical phenomena such as aurora, airglow, radio noise, magnetic field fluctuations and weather. More recently, perturbation of the ionosphere by space vehicles and atomic weapons tests have emphasized the need for studies of localized regions of the ionosphere. Thus it can be seen that a knowledge of the properties of the ionosphere is very important for many activities.

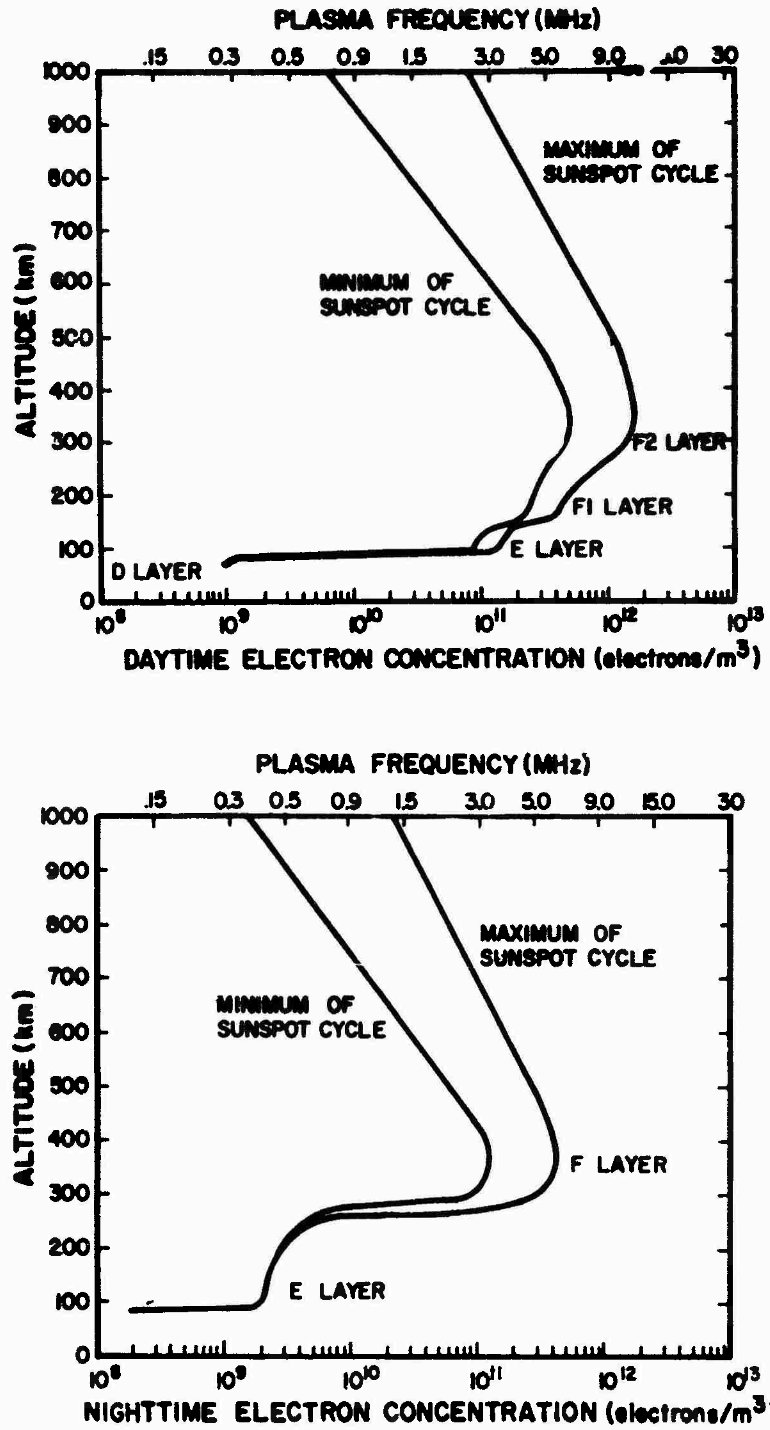


Fig. 1. Normal electron distribution of the ionosphere.

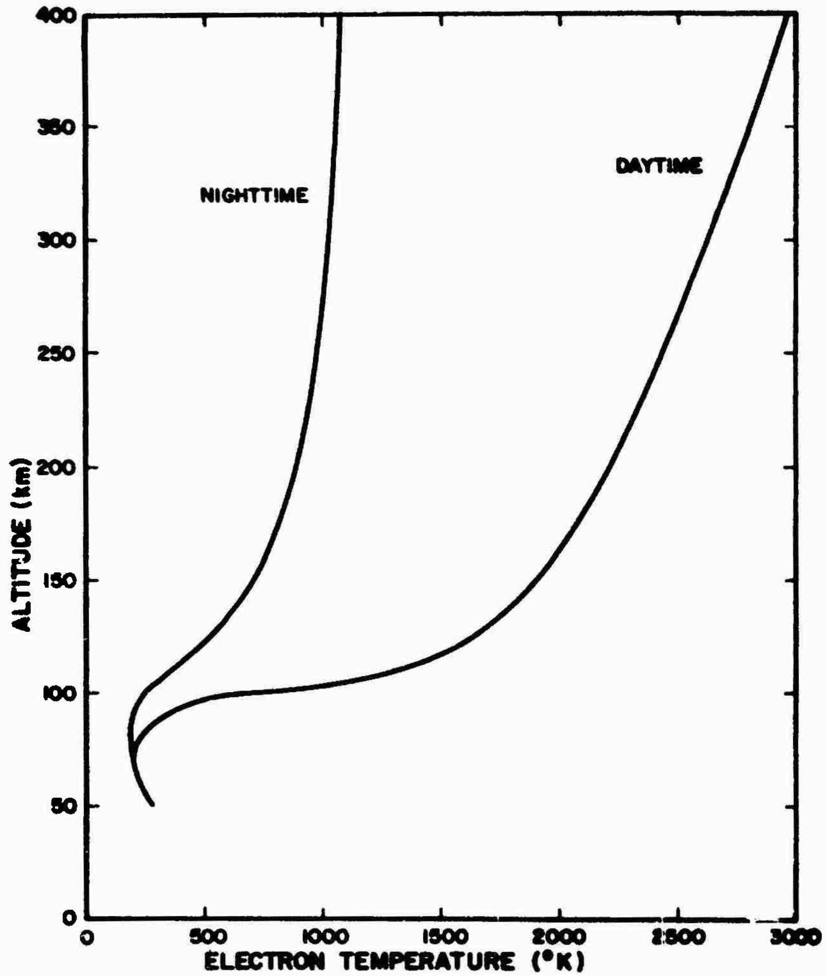


Fig. 2. Normal electron temperature in the ionosphere.

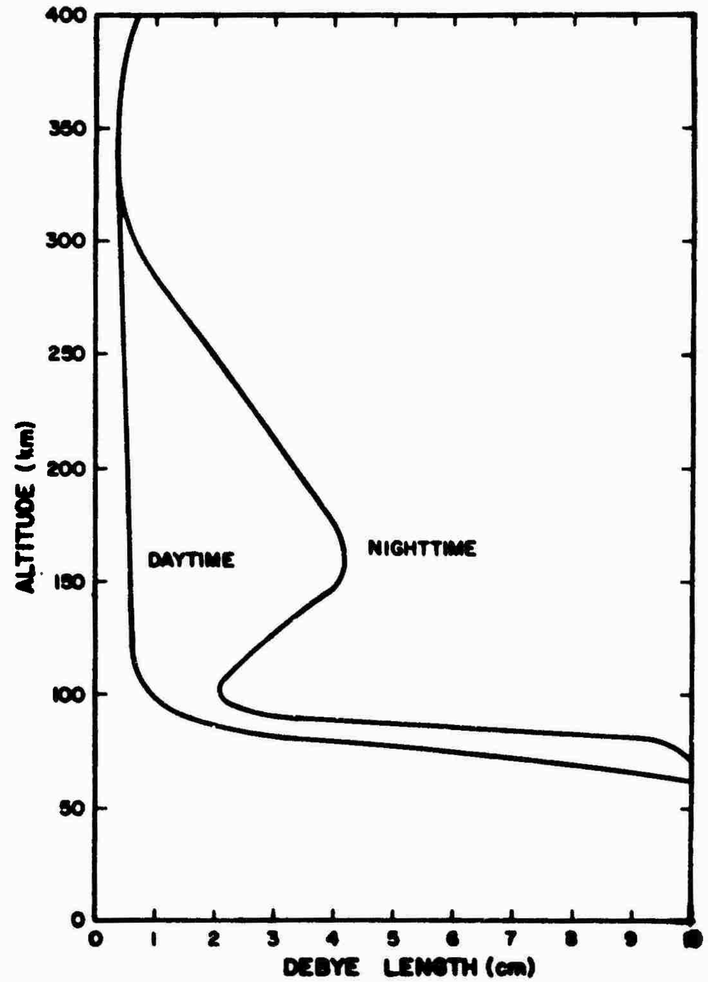


Fig. 3. Normal Debye length in the ionosphere.

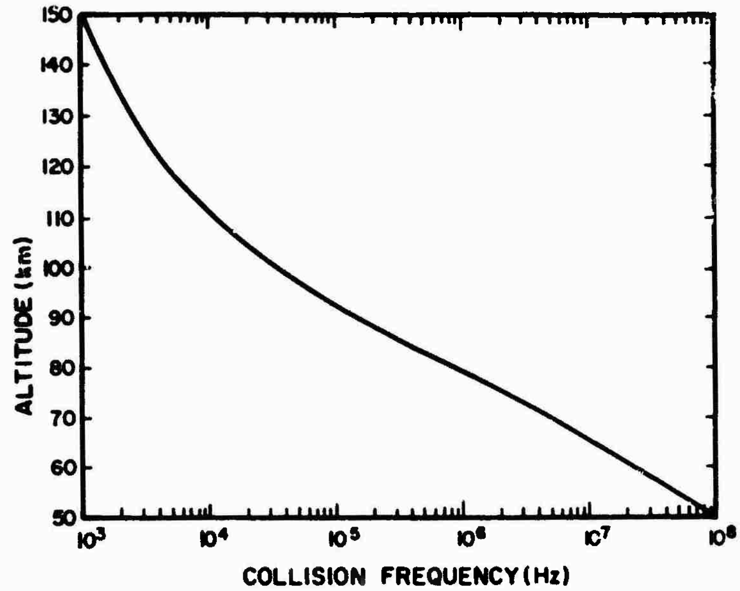


Fig. 4. Normal electron collision frequency in the ionosphere.

Ionospheric Measurement Techniques

Ionospheric parameters have been traditionally measured from the earth's surface by several techniques, notably by ionospheric sounders [*Mitra*, 1952, p. 219]. However, fine structure of localized regions and disturbed areas of the ionosphere can only be examined by direct, *in situ* measurements. Sounding rockets are often the most useful for examining these localized or disturbed regions. Satellite vehicles are, on the other hand, especially useful in obtaining worldwide coverage, often over an extended period of time.

Three general plasma measurement techniques have been extensively exploited for rocket and satellite application. The first technique was developed from the concepts of the ground-based ionospheric sounder. That is, the electromagnetic propagation characteristics of the ionospheric plasma are measured by transmitting probing signals between the carrier vehicle and the ground. Many variations of this basic technique

have been utilized even though extensive instrumentation and complex analyses are required.

The second technique is the application of the Langmuir Probe [*Langmuir*, 1923] to ionospheric measurements. One or more electrodes are extended from the vehicle into the ionospheric plasma and excited by various potentials. The resulting dc currents carry the measurement information. Related devices such as ion traps and resonance probes have also been developed. These all use dc potentials to collect and examine the ionized plasma particles by measuring dc currents from the collector electrodes. These techniques have nearly dominated the measurements of electron and ion temperatures and ion densities. These methods often suffer from disturbing vehicle potential perturbations; nevertheless, they have provided the only direct measurements of electron temperature.

The third method is the measurement of the plasma perturbated, RF probe impedance. Although previously well known in the laboratory, ionosphere measurements using this technique seem to have started when apparent antenna impedance changes were noted in a dipole being used in a rocket propagation experiment. This effect was then used to estimate the local electron density. Since that time, many variations of the basic technique have been used. The interaction of an antenna with the ionospheric plasma is the prime concern here, thus we will examine some of these RF impedance type experiments in details. Later, flight data from some of these experiments will be analyzed by applying the theory developed herein.

Three RF probes will be considered. Each technique utilizes a pair of 2.5 to 3-meter probes, sometimes referred to as a dipole antenna, mounted normal to the longitudinal vehicle axis and extended into the iono-

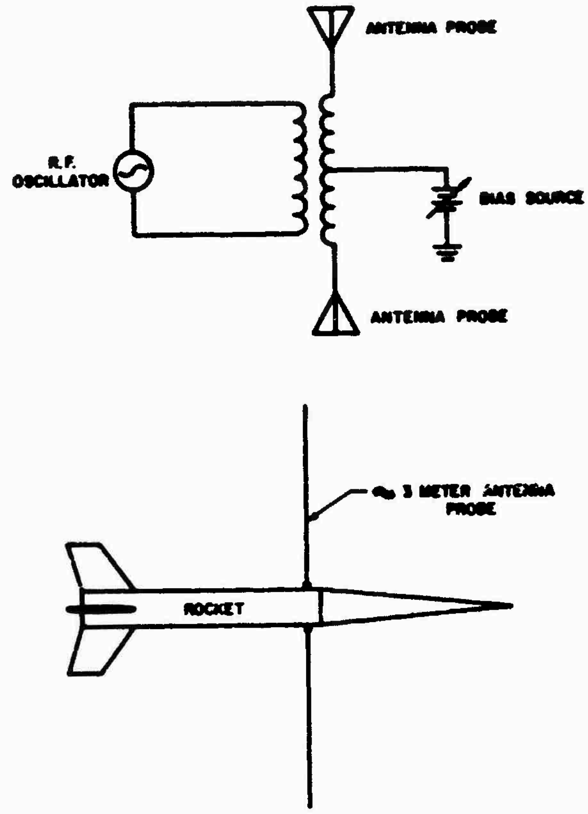


Fig. 5. Basic concept of RF probe systems.

spheric plasma (see Figure 5). In each case, the antenna is excited with a low-level RF signal (approximately 0.5 volt rms) in the 0.5 to 15-MHz range; in addition, a slowly varying bias voltage from 0 to +5 volts is placed on the antennas with respect to the vehicle skin. The bias modifies the sheath and allows the effect of the sheath to be determined. An analysis of the effects of the plasma on the antenna RF impedance or on the dc current variations as a function of frequency provides a measure of the plasma parameters.

Standing Wave Impedance Probe (SWIP)

Perhaps one of the most successful impedance probes is the Standing Wave Impedance Probe [*Ulwick et al.*, 1964]. This system measures the RF impedance of a dipole antenna which is being driven by one or two time-

multiplexed RF frequencies (see Figure 6). The impedance probe frequencies are set so that one or the other is near the local plasma frequency throughout the experiment. The RF fields from the antenna excite motion in the plasma electrons which, in turn, modify the antenna impedance. This change of impedance is indicated by a change in the standing wave pattern of the voltage along an artificial transmission line connecting the driving RF oscillator and the antenna. The voltage standing wave pattern is sampled, detected and telemetered to a ground station where it is recorded for further analysis. This experiment can provide reliable impedance measurements at one or two fixed exciting frequencies.

A very simple analysis is usually employed [*Ulwick et al.*, 1964], and this generally produces good results [*Baker, et al.*, 1966]. Occasionally, electron temperature and other effects become important, thus pointing out the need for further theoretical development [*Baker et al.*, 1965, and *Whale*, 1962].

Plasma Frequency Probe (PFP)

The Plasma Frequency Probe [*Haycock and Baker*, 1962] excites the antenna with a variable frequency that sweeps from about .1 to 10 MHz (see Figure 7). The resonance frequencies of the antenna impedance are sensed by determining the frequency where the phase angle between the RF antenna current and voltage is zero. A parallel resonance condition (high impedance) is expected to occur near the natural frequency of the plasma, that is, the plasma frequency. Experimentally, it has been found that a series resonance condition (low impedance) also occurs [*Baker et al.*, 1966]. It will be shown later that this series resonance is related to the electron temperature of the plasma.

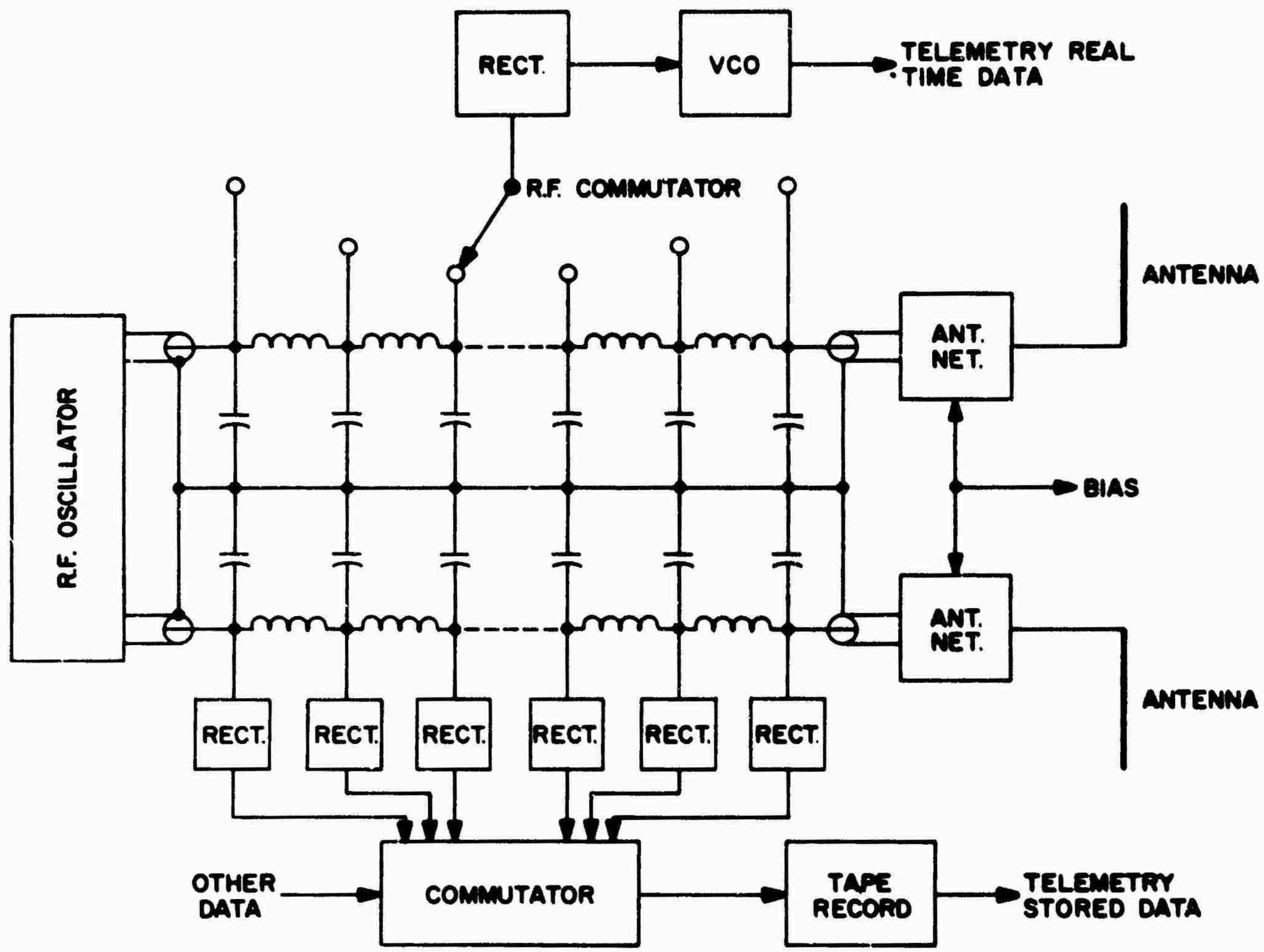


Fig. 6. Standing Wave Impedance Probe.

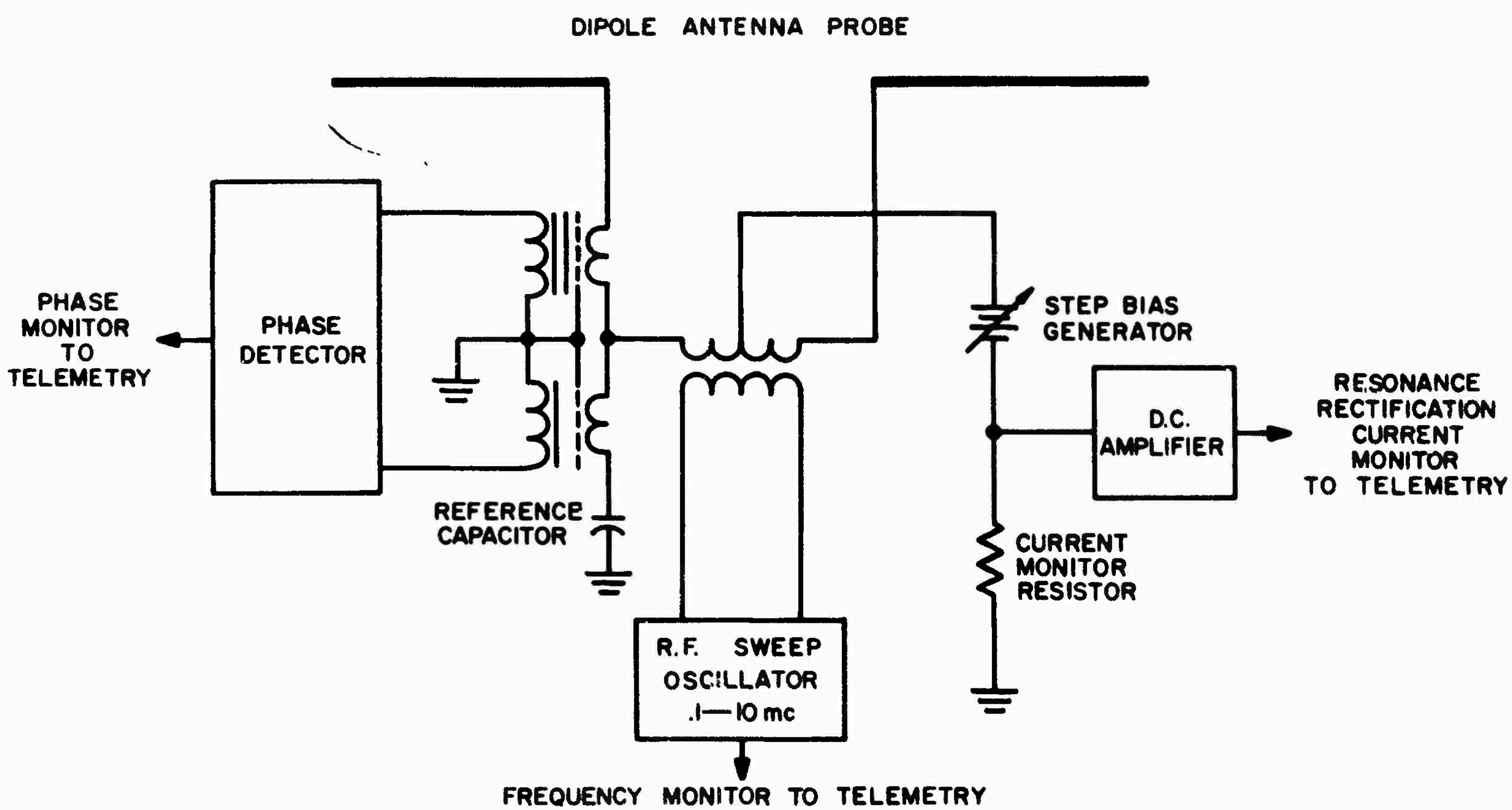


Fig. 7. Plasma Frequency and Resonance Rectification Probe.

## Resonance Rectification Probe (RRP)

The Resonance Rectification Probe [*Takayama, Ikegami and Miyazaki*, 1960; *Despain*, 1964] is a combination of a Langmuir Probe and an RF sweep oscillator similar to that used in the Plasma Frequency Probe (see Figure 7). A dc bias (0 to +5 volts), stepped with successive RF oscillator sweeps, is applied to the antenna. The resulting dc current is monitored. Because the RF antenna impedance changes with frequency and the dc characteristics of the antenna probe exhibit nonlinearities, the dc current is a function of the RF frequency. It had been contended that the frequency where the dc current reaches a maximum is the local electron plasma frequency [*Takayama*, 1960; *Miyazaki et al.*, 1960; *Ichikawa and Ikegami*, 1962; *Hirao and Muraoka*, 1964]; however, *Baker, Despain and Ulwick* [1966] have shown experimentally that this conclusion is not true; in fact, the maximum is more closely associated with the series resonance frequency. Thus additional theoretical work on antenna impedance is also needed to aid in the interpretation of this experiment.

## Preliminary Conclusion

The ionosphere is of general interest and exhibits interesting and complex behavior. A knowledge of the ionospheric parameters, especially in localized regions, is prerequisite to an understanding of this behavior. Some of the probe techniques for ionospheric parameter measurement require an analysis of the interaction of an RF-driven probe with the surrounding ionospheric plasma. The theories of such interactions will now be discussed in Chapter II.

# Chapter II

## PROBE THEORIES

The problem of interaction of a small RF excited probe with a plasma has been studied for at least sixty years [*Wilson and Gold*, 1906]. Most investigators have assumed that the impedance change is due solely to a change in the effective complex permittivity $\epsilon$ of a locally homogeneous ionized gas as expressed in the Appleton-Hartree Theory [*Ratcliffe*, 1962]; that is, the electrons are considered as mobile charges, free to move under the influence of electromagnetic fields. The electric current that results from such movement is accounted for by a change in the permittivity of the medium. If thermal motions and magnetic fields are ignored, the result of such a calculation is shown in Appendix A to produce the following expression for the total antenna impedance $Z_{at}$.

$$Z_{at} = \frac{1}{j\omega C_o}\left[1 - \frac{X}{1-jZ}\right]^{-1} + R_r\left[1 - \frac{X}{1-jZ}\right]^{-1/2} \tag{7}$$

where

$C_o$ = free space capacitance of the probe

$\omega$ = radian operating frequency

$\omega_N$ = radian plasma frequency

$\nu$ = electron collision frequency

$R_r$ = free space radiation resistance

$X$ = $\omega_N^2/\omega$

$Z$ = $\nu/\omega$

For $R_r$ and $\nu$ equal to zero, a particularly simple interpretation results in the probe being considered as a capacitor C of value

$$C = C_o \left[1-X\right] = C_o \left[1 - \frac{\omega_N^2}{\omega^2}\right] \tag{8}$$

This simple, idealized model has been extensively applied to interpret impedance probe results [*Ulwick et al.*, 1964; *Jackson and Pickar*, 1957; *Whale* 1963].

A more rigorous approach by *King* and *Harrison* [1960] results in some highly involved expressions; while an expression developed by *DeChamps* [1961] is somewhat simpler. However, neither of these works consider magnetic field or electron temperature effects and hence add little to the general formulation that we seek here. If magnetic field effects are to be considered, then the dielectric constant becomes a tensor [*Ratcliffe*, 1962] and the geometry of the probe must be considered. The prime interest of this paper is cylindrical geometry; therefore, only solutions for this case are considered.

Perhaps the most obvious approach for a tensor dielectric constant is to apply the tensor [ε] as given by the usual Appleton-Hartree formulation [*Ratcliffe*, 1962] or the formulation of *Sen* and *Wyler* [1960]. The Sen-Wyler approach allows for collisional frequency variations with electron energy but is otherwise identical to the Appleton-Hartree tensor. *Shkarofksy* [1961] has generalized the Appleton-Hartree equations even further by considering the degree of plasma ionization as well.

The question of how a given [ε] should be applied to determine antenna impedance has been considered by several workers. *Herman* [1963] and *Crouse* [1964] investigated conical geometries, while *Kaiser* [1962] considered biconical antennas. These workers all assumed a simple capacitor model. In Appendix A, the impedance of a cylindrical monopole,

using the capacitor model and the Appleton-Hartree formulation for $[\epsilon]$ is also developed. The result is

$$Z_a = \frac{1}{j\omega C_o}\left[1 + X\ \frac{1 - 1/2\ (Y/U)^2\ \sin^2\gamma}{U((Y/U)^2-1)}\right]^{-1} \tag{9}$$

where

$U = 1 - jZ$

$Y = \omega_H/\omega$

$\gamma$ = Angle between antenna axis and the magnetic field

The capacitor models discussed above ignore source current distributions and, in fact, all magnetic fields except the steady field as it appears in $[\epsilon]$. The classical approach to antenna theory, on the other hand, generally assumes a source current distribution and all the parameters are then determined from this specification. *Bramley* [1962] used this approach to first obtain results in an isotropic region similar to those of *King, Harrison and Denton* [1961]. Then he extended the results to the magneto-ionic case by deriving an approximation valid for weak magnetic fields or small electron densities. *Ament et al.* [1964] considered the general case and produced an expression for impedance that requires extensive numerical integration and, in general, seems to be very difficult to apply. *Balmain* [1964], however, has developed some very useful impedance relations. His results appear to be the most physically satisfying of all the work mentioned above and yet are quite tractable in terms of computation. His expression is

$$Z_a = \frac{a}{j\omega 2\pi\epsilon_o LK'\sqrt{F}}\left[\ln\frac{L}{R} - 1 - \ln\frac{a+\sqrt{F}}{2F}\right] \tag{10}$$

where

$$F = \sin^2\gamma + a^2\cos^2\gamma$$

$$a^2 = K'/K_o$$

$$K' = 1 - \frac{XU}{U^2-Y^2}$$

$$K_o = 1 - \frac{X}{U}$$

None of the above derivations includes the explicit effects of finite electron temperature. That is, a cold, yet lossy medium is always assumed. However, electron temperature can appreciably perturbate the impedance of an antenna probe in a plasma, other than its effect on collisional frequency. *Whale* [1963] observed such effects in an impedance probe experiment, primarily as an energy loss mechanism. It will also be shown later that the experimentally determined antenna series resonance frequencies of *Baker, Despain and Ulwick* [1966] are indeed related to the electron temperature as they suspected. Thus electron temperature must be considered in a derivation of antenna impedance.

Several workers have attempted to include electron temperature as a parameter. *Cohen* [1961] presented a thorough theoretical discussion of sources in isotropic warm plasmas. *Whale* [1963[ developed some theory to explain his experimental results mentioned above. *Balmain* [1964] applied Cohen's equations to his problem for the case of no magnetic field and derived an approximate impedance relation. In a similar manner, *Fejer* [1964] theoretically investigated spherical geometry without magnetic field and predicted a series resonance below the plasma frequency directly dependent upon the electron temperature, *Despain* [1965], in preliminary work to this report, investigated cylindrical geometry with similar results. The work of *Wait* [1964] should also be mentioned as it concerns slot antennas in warm plasmas. An attempt to develop a permittivity tensor [ε]

valid for finite electron temperature and to apply this [ε] to the capacitor model of the antenna met with failure (see last section of Appendix A). The impedance expression that resulted did not agree even approximately with the other theoretical work nor did it exhibit behavior known experimentally to be characteristic of such systems. Further discussion is included in Appendix A.

In reviewing the work mentioned above, it is apparent that further theoretical work is needed on the electron temperature effects on cylindrical antennas, especially in the presence of magnetic fields and electron collisions. Therefore, the present investigation was initiated in order to relate electron density, temperature and collisional frequency to antenna impedance.

## Chapter III

## DERIVATION OF ANTENNA IMPEDANCE

### Introduction

The basic assumption of this derivation is that the electrons behave as a continuous fluid and the effect of all electron interactions may be represented by various forces on the electron fluid. This view of a plasma is similar to that of *Spitzer* [1962], *Cohen* [1961] and *Fejer* [1964]. This approach allows the electron temperature effects to be taken into account as a property of the electron fluid. Besides the effect of electron density and temperature, the effects of an external constant magnetic field and electron collisions are also included. It is further assumed that the undisturbed plasma is neutral and homogeneous. Later some restricted space variations of the plasma parameters are allowed near the antenna probe surface by using a simple sheath model.

The antenna probe is taken to be an isolated, electrically short cylinder, whose length to radius ratio is large enough so that end effects can be ignored. Because of the long wavelength of the excitation, electromagnetic effects are assumed to be small in comparison to the plasma effects, and thus the antenna impedance may be calculated from the effective quasi-static electric fields of the antenna. It is further assumed that the excitation level is very low so that linearized equations can be used and only first order effects considered. *Exp* ($j\omega t$) time dependence and MKS units are assumed throughout.

The strategy of the derivation is to first develop a force equation that relates all the forces on an element of the electron fluid. From this equation, the velocity of the electron fluid is found. The velocity expression is

then substituted into the continuity equation and a second order partial differential equation is derived in terms of the scalar parameters n (electron density variation) and $\Phi$ (electric potential). Poisson's equation, also a second order partial differential equation in $\Phi$ and n, is used to simplify the first equation. A solution of this equation for n leads directly to a solution of Poisson's equation. Boundary conditions can be applied to determine a solution for the potential $\Phi$. Once $\Phi$ is known, the fields surrounding the antenna can be easily determined, and the antenna impedance can be found as the complex ratio of antenna potential to antenna current. Sheath effects are then added as an additional effect.

## Fundamental Equations

Consider the forces that are exerted on a volume element dV of the electron fluid. The force $F_p$ due to the variations of the pressure p is

$$d\vec{F}_p = -(\nabla p)dV \tag{11}$$

The Lorentz force $F_L$ on the element is

$$d\vec{F}_L = -Ne[\vec{E} + \vec{v} \times \vec{B}]\, dV \tag{12}$$

where

$N$ = average electron density (constant)

$e$ = electronic charge

$\vec{v}$ = average velocity of the electrons in the element

$\vec{B}$ = external constant magnetic field

$\vec{E}$ = electric field

$p$ = pressure variation from its mean value

and the force due to electron collisions is given in terms of the effective electron collision frequency $\nu$ as follows

$$d\vec{F}_c = -\nu Nm\,\vec{v}\,dV \tag{13}$$

where m is the electronic mass.

Applying Newton's Law, we equate these forces to the mass of the element times its acceleration to produce the force equation

$$Nm\,\frac{d\vec{v}}{dt}\,dV = -(\nabla p)dV - Ne[\vec{E} + \vec{v} \times \vec{B}]\,dV - \nu Nm\vec{v}dV \tag{14}$$

or

$$j\omega Nm\vec{v} = -\nabla p - Ne\,[\vec{E} + \vec{v} \times \vec{B}] - \nu Nm\vec{v} \tag{15}$$

Now $\nabla p$ may be written according to the chain rule of differentiation as

$$\nabla p = \frac{\partial p}{\partial \rho}\,\nabla \rho \tag{16}$$

where $\rho$ is the mass density. The adiabatic law relates p and $\rho$

$$p = C'\rho^{\gamma'} \tag{17}$$

where $\gamma'$ = ratio of specific heats. As discussed in Chapter I, $\gamma'$ has the value of 3.0 for an electron gas provided $\omega >> \nu$.

Now

$$\frac{\partial p}{\partial \rho} = 3C'\rho^2 \tag{18}$$

$$= 3p/\rho \tag{19}$$

but from the equation of state

$$p = KTn \tag{20}$$

where

$K$ = Boltzmann's constant

$T$ = electron temperature

$n$ = variation of electron density from its mean value,

and by definition

$$\rho = mn \tag{21}$$

thus

$$\frac{\partial p}{\partial \rho} = 3KT/m \tag{22}$$

At this point, it is interesting to note that $\frac{\partial p}{\partial \rho}$ is just the square f the propagation velocity $\mu$ that describes the velocity of longitudinal (sound) waves in compressible fluids. Thus

$$p = \mu^2 \nabla(mn) \tag{23}$$

where

$$\mu^2 = 3KT/m$$

Thus (15) is, in linearized parameters

$$j\omega Nm\vec{v} = -\mu^2 \nabla(mn) - Ne\,[\vec{E} + \vec{v} \times \vec{B}] - \nu Nm\vec{v} \tag{24}$$

or

$$N[j\omega\vec{v} + \omega_H \vec{v} \times \hat{\zeta} + \nu\vec{v}] = -\mu^2 \nabla n + (Ne/m)\ \nabla\Phi \tag{25}$$

where

$\omega_H$ = $eB/m$, electron gyrofrequency

$\hat{\zeta}$ = direction of magnetic field (unit vector)

$\Phi$ = scalar potential

Up to this point, the force equation is valid in any system of coordinates. A further attack on the problem will now require that the cylindrical coordinates $\hat{r}$, $\hat{\theta}$ and $\hat{z}$ be applied. The axis of the antenna probe has been chosen to coincide with the $\hat{z}$ axis (see Figure 8).

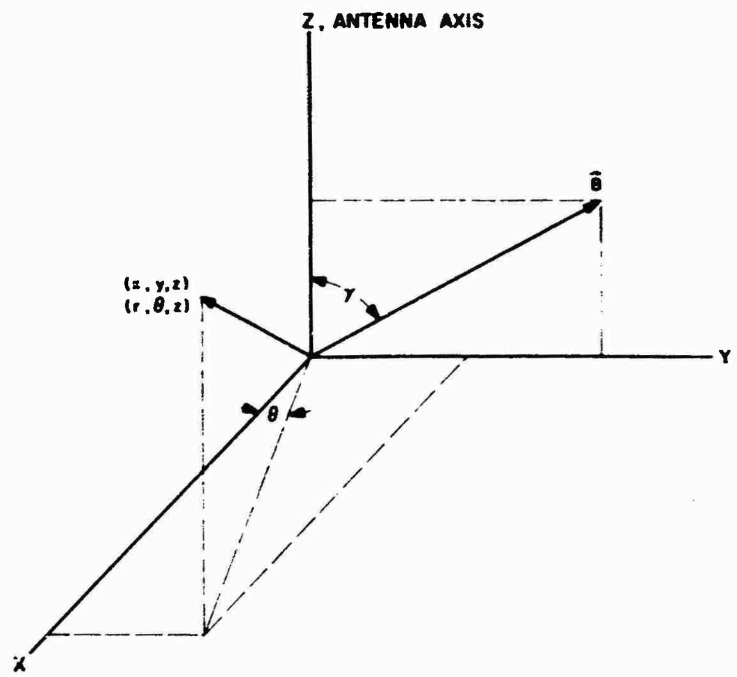


Fig. 8. Coordinate system.

The next step is to determine the velocity v in terms of n and $\Phi$. Because v is entangled in a curl operation, this is not a straight forward process. We begin by solving for the velocity components separately. In the r direction,

$$j\omega N\,[Uv_r - jY(v_\theta\zeta_z - v_z\zeta_\theta)] = -\mu^2 \frac{\partial n}{\partial r} + \frac{Ne}{m}\frac{\partial \Phi}{\partial r} \tag{26}$$

In the $\theta$ direction

$$j\omega N\,[Uv_\theta - jY(v_z\zeta_r - v_r\zeta_z)] = -\mu^2 \frac{1}{r}\frac{\partial n}{\partial \theta} + \frac{Ne}{m}\frac{1}{r}\frac{\partial \Phi}{\partial \theta} \tag{27}$$

In the z direction

$$j\omega N\,[Uv_z - jY(v_r\zeta_\theta - v_\theta\zeta_r)] = -\mu^2 \frac{\partial n}{\partial z} + \frac{Ne}{m}\frac{\partial \Phi}{\partial z} \tag{28}$$

where

$$Y = \omega_H/\omega$$

$$U = 1 - jZ$$

$$Z = \nu/\omega$$

However because of symmetry

$$\frac{\partial n}{\partial z} = 0 \quad \text{and} \quad \frac{\partial \Phi}{\partial z} = 0$$

and thus from (28)

$$v_z = jW(v_r\zeta_\theta - v_\theta\zeta_r) \tag{29}$$

or

$$v_z = jv_rW_\theta - jv_\theta W_r \tag{30}$$

and (26) and (27) become

$$j\omega UN\,[v_r - jW_z v_\theta + jW_\theta(jv_r W_\theta - jv_\theta W_r)] = -\mu^2 \frac{\partial n}{\partial r} + \frac{Ne}{m}\frac{\partial \Phi}{\partial r} \tag{31}$$

$$j\omega UN\,[v_\theta + jW_z v_r + jW_r(jv_r W_\theta - jv_\theta W_r)] = -\mu^2 \frac{1}{r}\frac{\partial n}{\partial \theta} + \frac{Ne}{m}\frac{1}{r}\frac{\partial \Phi}{\partial \theta} \tag{32}$$

where

$$W = Y/U$$

$$W_r = W\zeta_r$$

$$W_\theta = W\zeta_\theta$$

$$W_z = W\zeta_z$$

or

$$v_r(1 - W_\theta^2) = j\beta_r - v_\theta(W_r W_\theta - jW_z) \tag{33}$$

$$v_\theta(1 - W_r^2) = j\beta_\theta - v_r(W_r W_\theta + jW_z) \tag{34}$$

where

$$\beta_r = \beta_n \frac{\partial n}{\partial r} - \beta_p \frac{\partial \Phi}{\partial r}$$

$$\beta_\theta = \beta_n \frac{1}{r}\frac{\partial n}{\partial \theta} - \beta_p \frac{1}{r}\frac{\partial \Phi}{\partial \theta}$$

$$\beta_n = \frac{\mu^2}{UN\omega}$$

$$\beta_p = \frac{e}{Um\omega}$$

now with the aid of the relation

$$W_r^2 + W_\theta^2 + W_z^2 = W^2 \tag{35}$$

explicit values of $v_r$ and $v_\theta$ can be obtained. By solving the equations simultaneously, we find that

$$v_r = j \frac{\beta_r (1 - W_r^2) - \beta_\theta (W_r W_\theta - jW_z)}{1 - W^2} \tag{36}$$

and

$$v_0 = -j \frac{\beta_r (W_r W_\theta + jW_z) - \beta_\theta [1 - W_\theta^2]}{1 - W^2} \tag{37}$$

Now

$$\nabla \cdot \vec{v} = \frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \tag{38}$$

Hence $\frac{\partial v_r}{\partial r}$, $\frac{\partial v_\theta}{\partial \theta}$ and $\frac{\partial v_z}{\partial z}$ must be calculated.

$$\frac{\partial v_r}{\partial r} = \left[\frac{j}{1+W^2}\right] \left[\frac{\partial \beta_r}{\partial r} (1 - W_r^2) - \frac{\partial \beta_\theta}{\partial r} (W_r W_\theta - jW_z)\right] \tag{39}$$

$$\frac{\partial v_\theta}{\partial \theta} = \left[\frac{-j}{1+W^2}\right] \left[\frac{\partial \beta_r}{\partial \theta} (W_r W_\theta + jW_z) + \beta_r \left(\frac{\partial W_r}{\partial \theta} W_\theta + W_r \frac{\partial W_\theta}{\partial \theta} + j \frac{\partial W_z}{\partial \theta}\right) - \frac{\partial \beta_\theta}{\partial \theta} (1 - W_\theta^2) - 2\beta_\theta \left(W_\theta \frac{\partial W_\theta}{\partial \theta}\right)\right] \tag{40}$$

$$\frac{\partial v_z}{\partial z} = 0 \tag{41}$$

But

$$\frac{\partial W_r}{\partial \theta} = +W_\theta \tag{42}$$

$$\frac{\partial W_\theta}{\partial \theta} = -W_r \tag{43}$$

$$\frac{\partial W_z}{\partial \theta} = 0 \tag{44}$$

Thus

$$\frac{\partial \beta_r}{\partial r} = \beta_n \frac{\partial^2 n}{\partial r^2} - \beta_p \frac{\partial^2 \Phi}{\partial r^2} \tag{45}$$

$$\frac{\partial \beta_r}{\partial \theta} = \beta_n \frac{\partial^2 n}{\partial r \partial \theta} - \beta_p \frac{\partial^2 \Phi}{\partial r \partial \theta} \tag{46}$$

$$\frac{\partial \beta_\theta}{\partial r} = \beta_n \frac{1}{r} \frac{\partial^2 n}{\partial r \partial \theta} - \beta_p \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} - \beta_n \frac{1}{r^2} \frac{\partial n}{\partial \theta} + \beta_p \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} \tag{47}$$

$$\frac{\partial \beta_\theta}{\partial \theta} = \beta_n \frac{1}{r} \frac{\partial^2 n}{\partial \theta^2} - \beta_p \frac{1}{r} \frac{\partial^2 \Phi}{\partial \theta^2} \tag{48}$$

and thus

$$\begin{aligned}
\nabla \cdot \vec{v} = \left[\frac{i}{1-W^2}\right] & \Big[ \left( \frac{1}{r} \beta_n \frac{\partial n}{\partial r} - \frac{1}{r} \beta_p \frac{\partial \Phi}{\partial r} \right) (1-W_r^2) \\
& - \left( \frac{1}{r^2} \beta_n \frac{\partial n}{\partial \theta} - \frac{1}{r^2} \beta_p \frac{\partial \Phi}{\partial \theta} \right) (W_r W_\theta - jW_z) \\
& + \left( \beta_n \frac{\partial^2 n}{\partial r^2} - \beta_p \frac{\partial^2 \Phi}{\partial r^2} \right) (1-W_r^2) \\
& - \left( \beta_n \frac{1}{r} \frac{\partial^2 n}{\partial r \partial \theta} - \beta_p \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} \right) (W_r W_\theta - jW_z) \\
& - \left( \beta_n \frac{1}{r} \frac{\partial^2 n}{\partial r \partial \theta} - \beta_p \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} \right) (W_r W_\theta + jW_z) \\
& + \left( \beta_n \frac{1}{r^2} \frac{\partial n}{\partial \theta} - \beta_p \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} \right) (W_r W_\theta - jW_z)
\end{aligned}$$

$$+(\beta_n \frac{1}{r^2} \frac{\partial^2 n}{\partial \theta^2} - \beta_p \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2})\ (1-W_\theta^2)$$

$$-(\beta_n \frac{1}{r} \frac{\partial n}{\partial r} - \beta_p \frac{1}{r} \frac{\partial \Phi}{\partial r})\ (W_\theta^2 - W_r^2)$$

$$-2(\beta_n \frac{1}{r^2} \frac{\partial n}{\partial \theta} - \beta_p \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta})\ (W_\theta W_r)] \tag{49}$$

This can be reduced to

$$\nabla \cdot \vec{v} = \left[\frac{j}{1-W^2}\right] [(\beta_n \nabla^2 n - \beta_p \nabla^2 \Phi) - W^2 \mathrm{Sin}^2\gamma(\beta_n S(n) - \beta_p S(\Phi))] \tag{50}$$

where

$$S(x) = \mathrm{Cos}^2\theta \frac{\partial^2 x}{\partial r^2} - 2\ \mathrm{Sin}\theta\ \mathrm{Cos}\theta \frac{1}{r} \frac{\partial^2 x}{\partial r \partial \theta} + \mathrm{Sin}^2\theta \frac{1}{r^2} \frac{\partial^2 x}{\partial \theta^2}$$

$$+ \mathrm{Sin}^2\theta \frac{1}{r} \frac{\partial x}{\partial r} + 2\ \mathrm{Sin}\theta\ \mathrm{Cos}\theta \frac{1}{r^2} \frac{\partial x}{\partial \theta} \tag{51}$$

This result can now be substituted into the continuity equation

$$N(\nabla \cdot \vec{v}) + j\omega n = 0 \tag{52}$$

and Poisson's equation

$$\nabla^2 \Phi = ne/\varepsilon_o \tag{53}$$

can also be applied to yield the result

$$N \left[\frac{j}{1-W^2}\right] \left[\frac{\mu^2}{UN\omega} \nabla^2 n - \frac{e}{Um\omega} (ne/\varepsilon_o) - W^2 \mathrm{Sin}^2\gamma(\beta_n S(n) - \beta_p S(\Phi))\right]$$

$$+ j\omega n = 0 \tag{54}$$

Simplifying

$$\nabla^2 n = -\alpha^2 n + W^2 \mathrm{Sin}^2\gamma[S(n) - U(\omega_N^2/\mu^2)(\varepsilon_o/e)\ S(\Phi)] \tag{55}$$

where

$$\alpha^2 = (\psi^2 - 1)\ \omega_N^2/\mu^2 \tag{56}$$

$$\psi^2 = (U^2 - Y^2)/UX \tag{57}$$

$$\omega_N^2 = Ne^2/m\varepsilon_o \tag{58}$$

$$X = \omega_N^2/\omega^2 \tag{59}$$

$$W = Y/U \tag{60}$$

For either $\gamma \ll 1$ or $W \ll 1$, this equation can be solved directly. However, the general case requires the solution for a pair of coupled, second order partial differential equations whose parameters are functions of the space coordinates. Clearly a straight forward analytic solution seems to be out of the realm of possibility, and it appears that an approximation technique will be required in order to solve the generalized system equations. In any case, the solution for $\gamma = 0$ is still of the highest interest since this only mildly restricts the applicability of the theory. If the restriction $\gamma = 0$ had been applied from the beginning, the derivation would have been a little less complicated; however by developing (55), we now have an equation that can possibly be solved at a later time to provide an estimate of antenna impedance for $\gamma \neq 0$. Thus for the purposes of this report, it will be sufficient to limit the solution and only solve the restricted equation (61).

$$\nabla^2 n = -\alpha^2 n \tag{61}$$

Thus the plasma may be described by the system equations (53) and (61). For $\gamma = 0$ however, symmetry exists in the $\theta$ direction so that only the r coordinate need be considered. Thus it is sufficient to solve the set of equations

$$\nabla_r^2 \Phi = ne/\varepsilon_o \tag{62}$$

$$\nabla_r^2 n = -\alpha^2 n \tag{63}$$

where the r subscript indicates that only the r component is under consideration.

Solution of System Equations

Equation (63) may be expanded and transformed by performing the indicated operations and allowing $x = \alpha r$.

$$\frac{d^2 n}{dx^2} + \frac{1}{x}\frac{dn}{dx} + n = 0 \tag{64}$$

This is the familiar Bessel's equation of zero order. Hence, it has, as solutions, all of the various Bessel functions of order zero. Since x is in general a complex variable, it is convenient to use a linear combination of the Hankel functions $H_o$ (x) and $H_o^{(2)}$ (x) [sometimes $H_o$(x) is written as $H_o^{(1)}$ (x)]. In general, there are two values of x to be considered, $x = -\sqrt{\alpha^2}$ r and $x = +\sqrt{\alpha^2}$ r. However, it is a property of the Hankel functions [*Jahnke and Emde*, 1945] that $H_o^{(2)}$ (-x) = $H_o$(x) and that $H_o$(-x) = $H_o^{(2)}$ (x). Thus we need to consider only the one solution of x which lies in the first two quadrants of the complex plane. Then for large values of x, that is, at distances far removed from the antenna, we require that the solution for n, the density variations, become small. Thus since $H_o^{(2)}$ (x) approaches infinity as x approaches zero, only H(x) can be an acceptable solution.

Therefore,

$$n = n_o H_o(\alpha r) \tag{65}$$

where

$n_o$ = arbitrary constant to be determined by boundary conditions

$\alpha$ = the square root of $\alpha^2$ that lies in the top half of the complex plane.

The solution to the homogeneous part of the system equation (62) may be written as

$$\Phi = C_1 \ln(r/L) \tag{66}$$

where $C_1$ is an arbitrary constant yet to be determined and L is the length of the antenna. This form of solution was chosen to produce the correct form of potential near the surface of a long, thin, charged rod in free space. Since (62) is very similar to (63), a solution of the form

$$\Phi = C_2 H_o(\alpha r) \tag{67}$$

would also be expected to satisfy (62). Such is indeed the case and the total solution may be written as

$$\Phi = C_1 \ln(r/L) - (en_o/\varepsilon_o\alpha^2)\, H_o(\alpha r) \tag{68}$$

where $C_2 = -n_o e/\varepsilon_o\alpha^2$ has been chosen so as to satisfy (62).

## Boundary Conditions

General solutions for n and $\Phi$ now exist; however, the arbitrary constants $C_1$ and $n_o$ have not yet been determined. These constants can be obtained by a consideration of boundary conditions near the antenna surface.

Our original pair of second order differential equations require the specification of four independent boundary conditions to determine n and $\phi$. Two of these conditions were fixed in the derivation above when both n and $\phi$ were required to approach zero as the distance from the antenna increased indefinitely. Only one of the other two conditions need be specified if only the antenna impedance is desired. This is true since we desire not a knowledge of $\phi$ or n but only a knowledge of the ratio of antenna voltage to current. Ultimately, one of the arbitrary constants $n_o$ or $C_1$ must cancel out of the impedance expression. Thus if a boundary condition at the antenna surface can be specified such that $C_1$ is determined, it will be possible to later cancel out $n_o$ and completely determine the antenna impedance.

It has previously been assumed that the plasma surrounding the antenna is homogeneous; that is, that no sheath exists. Immediately inside the antenna-plasma boundary, the electrons are no longer in a free, gaseous state and are thus essentially restrained from any movement. Hence, it is assumed that the velocity v is zero at the antenna surface.

This is the same boundary condition used by *Fejer* [1964] in a similar problem. As he points out, it does not correspond exactly to the physical situation, but only approximates it. More exact boundary conditions can become very difficult, but they can also lead to a more general solution. *Balmain* [1965], for example, has considered the complex boundary conditions that actually exist at a probe-plasma boundary, and it is possible that his technique could be applied in this case. However, it is not necessary to include all of these complex conditions if only high frequency, small amplitude effects are of interest as is the case in this derivation.

For v = 0 at the antenna surface (r = R), equation (25) becomes at the surface

$$[\mu^2 \nabla_r n]_{r=R} = [(Ne/m)\ \nabla_r \Phi]_{r=R} \tag{69}$$

$$\frac{\mu^2 e}{\varepsilon_o}\left[\frac{\partial n}{\partial r}\right]_{r=R} = \omega_N^2 \left[\frac{\partial \Phi}{\partial r}\right]_{r=R} \tag{70}$$

but from (68) and the fact that $\frac{d}{dx}\ [H_o(x)] = -H_1(x)$

$$\frac{\partial \Phi}{\partial r} = C_1/r + (en_o/\varepsilon_o \alpha)\ H_1(\alpha r) \tag{71}$$

and similarly

$$\frac{\partial n}{\partial r} = -n_o \alpha H_1(\alpha r) \tag{72}$$

Thus

$$-\mu^2 (e/\varepsilon_o)(n_o \alpha H_1(\alpha R)) = \omega_N^2\ (C_1/R + (en_o/\varepsilon_o \alpha) H_1(\alpha R)) \tag{73}$$

or

$$C_1 = -(en_o R/\varepsilon_o \alpha) H_1(\alpha R)\ \psi^2 \tag{74}$$

since

$$\psi^2 = 1+\mu^2 \alpha^2/\omega_N^2 \tag{75}$$

Thus the expression for the potential, (68), becomes

$$\Phi = -(n_o e/\varepsilon_o \alpha^2)\ H_o\ (\alpha r) + \alpha R \psi^2\ H_1\ (\alpha R)\ \ln\ (r/L) \tag{76}$$

## Antenna Impedance

Impedance is defined as the complex ratio of voltage to current. In the case of the antenna under consideration here, the voltage is just the potential at the surface of the antenna and the current is just the displacement current summed over the antenna surface. Thus

$$Z_a = \frac{\Phi(R)}{\iint_S \vec{J}_D \cdot d\vec{S}} \tag{77}$$

or since

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = j\omega\vec{D} \tag{78}$$

$$Z_a = \frac{\Phi(R)}{2\pi RLj\omega\varepsilon_o E_r(R)} \tag{79}$$

but

$$E_r(R) = -\left[\frac{\partial\Phi}{\partial r}\right]_{r=R} \tag{80}$$

and

$$\frac{\partial\Phi}{\partial r} = (n_o e/\varepsilon_o \alpha^2)(\alpha H_1(\alpha r) \;\; - \alpha R\psi^2 H_1(\alpha R)/r) \tag{81}$$

Thus (81) becomes

$$Z_a = \frac{1}{j\omega 2\pi RL\varepsilon_o} \; \frac{H_o(\alpha R) - \alpha R\psi^2 H_1(\alpha R)\ln(L/R)}{\alpha H_1(\alpha R) - \alpha R\psi^2 H_1(\alpha R)/R} \tag{82}$$

or

$$Z_a = \frac{\ln(L/R)}{j\omega 2\pi\varepsilon_o L} \; \frac{H_o(\alpha R)/\alpha RH_1(\alpha R) \;\; \ln(L/R) \; - \; \psi^2}{1 - \psi^2} \tag{83}$$

Now the free space capacitance of a long thin rod is easily shown to be

$$C_o = 2\pi\epsilon_o L/\ln(L/R) \tag{84}$$

Thus

$$Z_a = \frac{-j}{\omega C_o}\left[\frac{\phi(\alpha R)\ /\ln(L/R)-\psi^2}{1-\psi^2}\right] \tag{85}$$

where

$$\phi(\alpha R) = H_o(\alpha R)/\alpha R H_1(\alpha R) \tag{86}$$

This is the impedance of a monopole antenna in a magneto-ionic plasma with the antenna axis aligned with the magnetic field.

The expression includes electron collision effects as a parameter in $\alpha$. If $\nu = 0$ and $X \geq 1$, then $\alpha$ becomes entirely imaginary and $\phi$ can be expressed in terms of the real modified Bessel functions of the second kind $K_o(x)$ and $K_1(x)$

Thus

$$\phi(\pm jx) = \frac{H_o(\pm jx)}{\pm jxH_1(\pm jx)} = \pm\frac{K_o(x)}{xK_1(x)} \tag{87}$$

We will find this result useful in some cases where $\nu = 0$.

If $\omega_H = 0$, then $\psi^2$ becomes

$$\psi^2 = U/X \tag{88}$$

while if $\nu = 0$ as well, then a particularly simple expression results

$$\psi = \omega/\omega_N \tag{89}$$

and

$$\alpha^2 = (\omega^2 - \omega_N^2)/\mu^2 \tag{90}$$

or

$$\alpha' = [(\omega_N^2 - \omega^2)/\mu^2]^{1/2} \tag{91}$$

where

$$\alpha'^2 = -\alpha^2 \tag{92}$$

This expression is useful when (87) is used to determine $\phi$ as

$$\phi(\alpha'R) = \frac{K_o(\alpha'R)}{\alpha'RK_1(\alpha'R)} \tag{93}$$

Sheath Effects

The first order effects of the plasma sheath on the antenna impedance can be taken into account by use of a very simple physical model. As long as the dc potential of the antenna is below the plasma potential (this is usually the case), a region of electron depletion will surround the antenna. The mechanism of this sheath formation is discussed in Chapter I. The model assumes that no electrons exist within a region of S Debye lengths of the antenna. Generally S will vary from 0 when the antenna is at the plasma potential, through about 4 when no net dc current is allowed to flow to the antenna, and up to about 10 if large negative potentials are applied to the probe. Thus the effective radius $R_S$ of the sheath is

$$R_S = R_A + S\lambda_D \tag{94}$$

The capacitance between the antenna and the surface of the plasma is then the capacitance of two concentric cylinders of radii $R_A$ and $R_S$.

$$C_S = 2\pi\varepsilon_o L/\ln(R_S/R_A) \tag{95}$$

The effective antenna impedance is then the impedance of this capacitor in series with the impedance of an antenna of radius $R_S$ since the plasma begins at a radius of $R_S$. The general impedance relation is then

$$Z_a = \frac{1}{j\omega C_S} + \frac{1}{j\omega C_o} \frac{\phi(\alpha R_S)/\ln(L/R_S)-\psi^2}{1-\psi^2} \tag{96}$$

where

$$C_o = 2\pi\varepsilon_o L/\ln(L/R_S) \tag{97}$$

The derivation of the antenna impedance is now complete. It can be seen that the major goals have been met; that is, the effects of a hot, lossy, magneto-ionic plasma on antenna impedance have been calculated. It should be noted that this expression is valid when the axis of the antenna is parallel to the magnetic field. A summary of the final expression and related parameters is tabulated in Table I.

TABLE I

Summary of Antenna Impedance Theory

| | | |
|---|---|---|
| Antenna Impedance | | $Z_a = \frac{1}{j\omega C_S} + \frac{1}{j\omega C_o}\left[\frac{\phi(\alpha R_S)/\ln(L/R_S)-\psi^2}{1-\psi^2}\right]$ |
| Sheath Capacitance | | $C_S = 2\pi\varepsilon_o L/\ln(R_S/R_A)$ |
| Antenna System Capacitance | | $C_o = 2\pi\varepsilon_o L/\ln(L/R_S)$ |
| Sheath Radius | | $R_S = R_A + S\lambda_D$ |
| Debye Length | | $\lambda_D = (K/4\pi^2 m)^{1/2}(T^{1/2}/f_N)$ |
| Antenna Length | $L$ | $\phi(\alpha R) = H_o(\alpha R)/\alpha R H_1(\alpha R)$ |
| Antenna Radius | $R_A$ | $\psi^2 = (U^2 - Y^2)/UX$ |
| Exciting Frequency (radian) | $\omega$ | $\alpha^2 = (\psi^2-1)\omega_N^2/\mu^2$ |
| Plasma Frequency (radian) | $\omega_N$ | $\mu^2 = 3KT/m$ |
| Collision Frequency | $\nu$ | $U = 1-jZ$ |
| Gyrofrequency (radian) | $\omega_H$ | $X = \omega_N^2/\omega^2$ |
| Electron Temperature | $T$ | $Y = \omega_H/\omega$ |
| Electronic Mass | $m$ | $Z = \nu/\omega$ |
| Boltzmann's Constant | $K$ | |
| Sheath Scaling Constant | $S$ | |

# Chapter IV

## COMPARISON OF RESULTS

### Introduction

It is desirable to empirically examine the impedance expression developed in Chapter III in order to compare it to both experimental data and other theories. Generally, other theoretical expressions are of completely different form and often either gross approximations or empirical methods must be applied before a favorable comparison can be discerned. Thus, the theoretical expression developed here and the other theoretical results of Chapter II have been programmed for digital computer analysis so that empircal comparisons can be carried out.

If the conventional free space expression for antenna impedance of a short dipole is examined [*Balmain, 1964, p. 56*], it is found that the impedance in the low frequency limit is just the impedance of the free space capacitance given by

$$C_o = 2\pi\varepsilon_o L/(\ln(L/R)-1) \tag{98}$$

This expression is the same as the $C_o$ derived in Chapter III except for the -1 term. This term represents the end effects that were ignored in the formulation of Chapter III. The above expression could be used to define $C_o$, but it is even more preferable to use an experimentally measured value of free space capacitance so that all perturbating influences can be taken into account (rocket body, etc.). Thus the actual mean antenna radius will be used in the impedance expressions, but the length of the antenna in the calculations will be changed from its true value to a value that will produce the correct value of $C_o$ as determined by measurements.

It could be argued that end effects could modify more than just the free space capacitance. These effects should be small, however, since the plasma medium tends to shield the antenna from any external disturbing influences at distances greater than a few Debye lengths (see Chapter I). This is especially true at frequencies below the plasma frequency and at frequencies above the plasma frequency, the end effects are not especially critical.

The empirical comparison of impedance theories can be approached from many directions. However, it is desirable to examine theoretical results so that they can easily be compared to experimental measurements as well. Thus the chosen antenna dimensions are typical of experimental antennas. In analogy with the Plasma Frequency Probe, the plasma parameters can all be held constant while the exciting frequency is swept through a wide range of values (.1 to 10 MHz) and the impedance determined. The theory can also be explored by fixing the exciting frequency and determining the antenna impedance as a function of the dominant plasma variable,plasma frequency with electron temperature as a parameter. This situation will then correspond to the Standing Wave Impedance Probe. It is also possible to plot antenna resistance versus antenna reactance with both plasma frequency and temperature as parameters. Perhaps only the dominate characteristics of the impedance are desired, such as the series and parallel resonance frequencies of the antenna. This situation actually more closely approximates the plasma frequency probe which determines only the frequencies of zero phase or resonance. Thus, these four methods will all be used to illustrate antenna impedance throughout the remainder of this paper.

A computer program, (SWP4MD) was written to plot the antenna impedance as a function of either plasma frequency or exciting frequency. Various subroutines were written to calculate the antenna impedance according to the

different theories of interest. Thus, empirical impedance plots of any of these theories could be obtained by use of the proper subroutine in the main computer program. Fortran listings of these programs are included in Appendix B for reference.

## Cold Plasma Comparison

It was mentioned earlier that most of the proposed theories of antenna impedance are for the cold plasma case. Thus the first test is to compare the derived impedance relations with other theories for zero electron temperature. For zero temperature, however, the theory of Chapter III reverts to the cold plasma magneto-ionic theory of Appendix A. This theory agrees very well with other results and is discussed and empirically compared to Balmain's Theory in Appendix A.

For non-zero electron temperature, the author is only aware of two other theories for cylindrical antenna impedance; that of *Balmain* [1964] and that of *Whale* [1965]. Both considered the case of finite electron temperature, but only for no magnetic field.

## Balmain Finite Temperature Theory

Balmain's theoretical result contained several difficult transendental functions, and he found it expedient to assume that the parameter $\alpha'R$ (see below) is large compared to one. This approximation greatly simplified his more exact result. The resulting expression is

$$Z_a = \frac{\ln(L/R)-1 + (K_o-1)/2\alpha'R}{j\omega 2\pi\varepsilon_o K_o L} \qquad (99)$$

where

$$K_o = 1-X/U$$

$$\alpha' = (X-1)^{1/2}\omega/\mu$$

## Whale Theory

*Whale* [1963], considered antenna power losses due to electroacoustic waves. Therefore, his results apply to an antenna in a warm plasma. His expression can be used to estimate antenna impedance if simple ionic theory is assumed for the antenna reactance while the power loss is interpreted as the result of antenna resistance. Whale's power loss equation is

$$Q_1 = \frac{X\omega\beta^2 V_a^2 (C_o/L)}{4\pi\epsilon_o R_s \sqrt{1-X}} \tag{100}$$

where

$Q_1$ = Radiated power per unit length

$\beta$ = A special function calculated by Whale and presented in the form of a graph

$V_a$ = Antenna voltage

$R_s$ = $R_a + 4\lambda_D$

If this expression is used to estimate antenna resistance and magneto-ionic theory is used to find the antenna reactance, the result is

$$Z_a = R_L/2 \pm [(R_L/2)^2 - X_a^2]^{1/2} - \frac{1}{\omega C_o(1-X)} \tag{101}$$

where

$$R_L = 2\pi\epsilon_o L R_s \sqrt{1-X}\,/X\beta^2 \omega C_o^2 \tag{102}$$

## Comparison of Theories

Both of the above expressions, as well as the expressions for antenna impedance from the Despain and magneto-ionic theories were programmed as computer subroutines with the name ANTZ. Thus all the theories could be simulated and later compared by using the desired subroutine in the main computer program.

Balmain's expression is plotted as a function of exciting frequency in Figure 9. Figure 10 is a similar plot of the Despain theory with the same parameters. For T = 0, the results from each theory are identical. However, as the temperature is increased, some discrepancies appear. These are especially evident in the antenna resistance when the exciting frequency is above the plasma frequency. Near T = 1000°K, there is reasonably good agreement, but as the temperature is increased further, it can be seen that the shapes of corresponding curves begin to show radical differences. This is very evident just below the plasma frequency. This discrepancy should not be unexpected since for high temperatures and for exciting frequencies near the plasma frequency, $\alpha'R$ becomes small and the large $\alpha'R$ approximation made by Balmain is not valid. Balmain's complete, though very difficult expression, must be used if his theory is to apply under these conditions.

It should be noted that in Balmain's theory (in contrast to the Despain theory) no appreciable reactance variation occurs as a function of temperature for exciting frequencies above the plasma frequency. This lack of variation is viewed with suspicion since even in regions where $\alpha'R$ is large, it seems only reasonable that antenna reactance is always changed to some degree by a change in electron temperature and antenna resistance. Appendix A also includes a discussion of a problem that occurs in Balmain's theory under certain special conditions.

It can thus be seen that the general characteristics of the two theories agree in this particular example for temperatures below 1000°K.

Figure 11 contains a comparison of the Balmain and Despain theories for fixed exciting frequency with plasma frequency as a variable. Again, good agreement is noted for low temperatures and for operating frequencies well separated from the plasma frequency. Again, some disagreement is evident wherever $\alpha'R$ becomes small. In fact, in the Balmain theory, the resistance

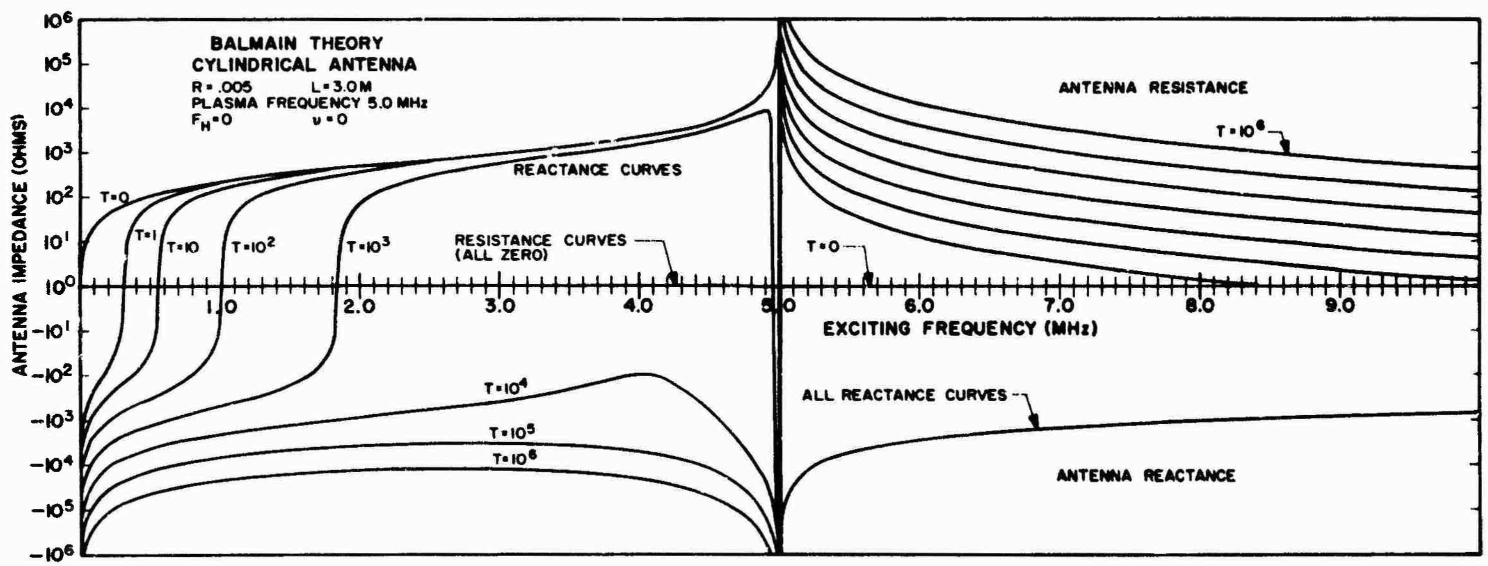


Fig. 9. Balmain Theory Antenna Impedance.

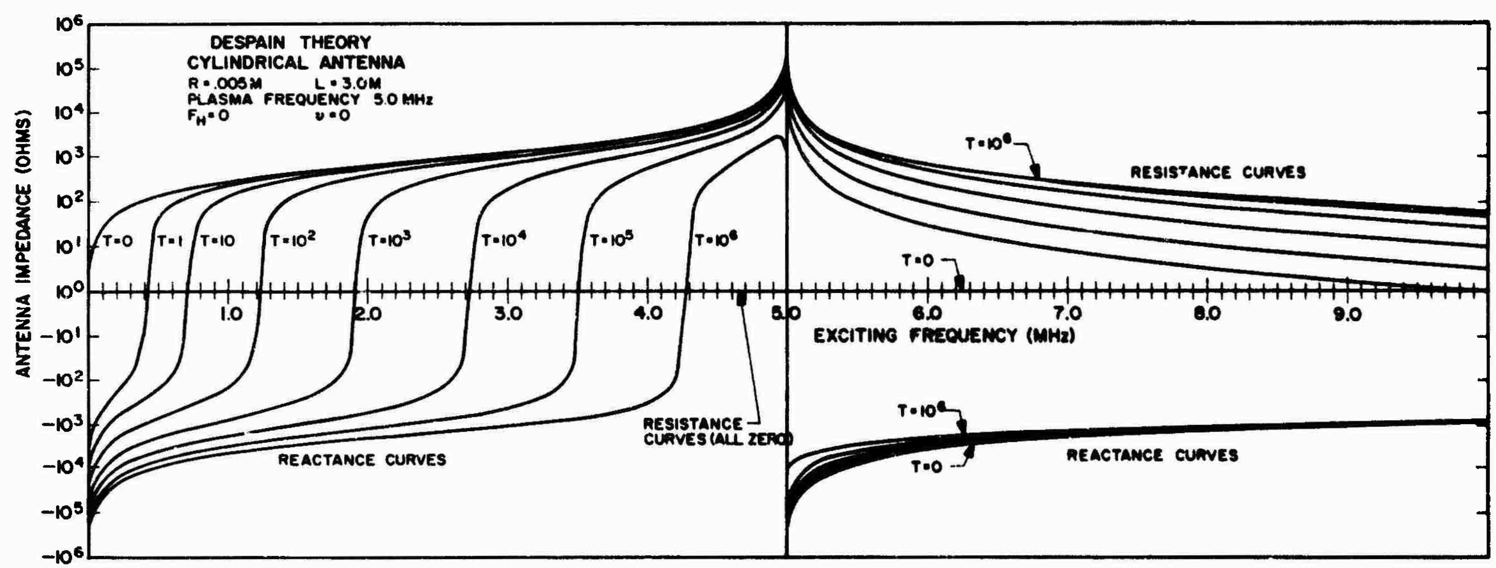


Fig. 10. Despain Theory Antenna Impedance.

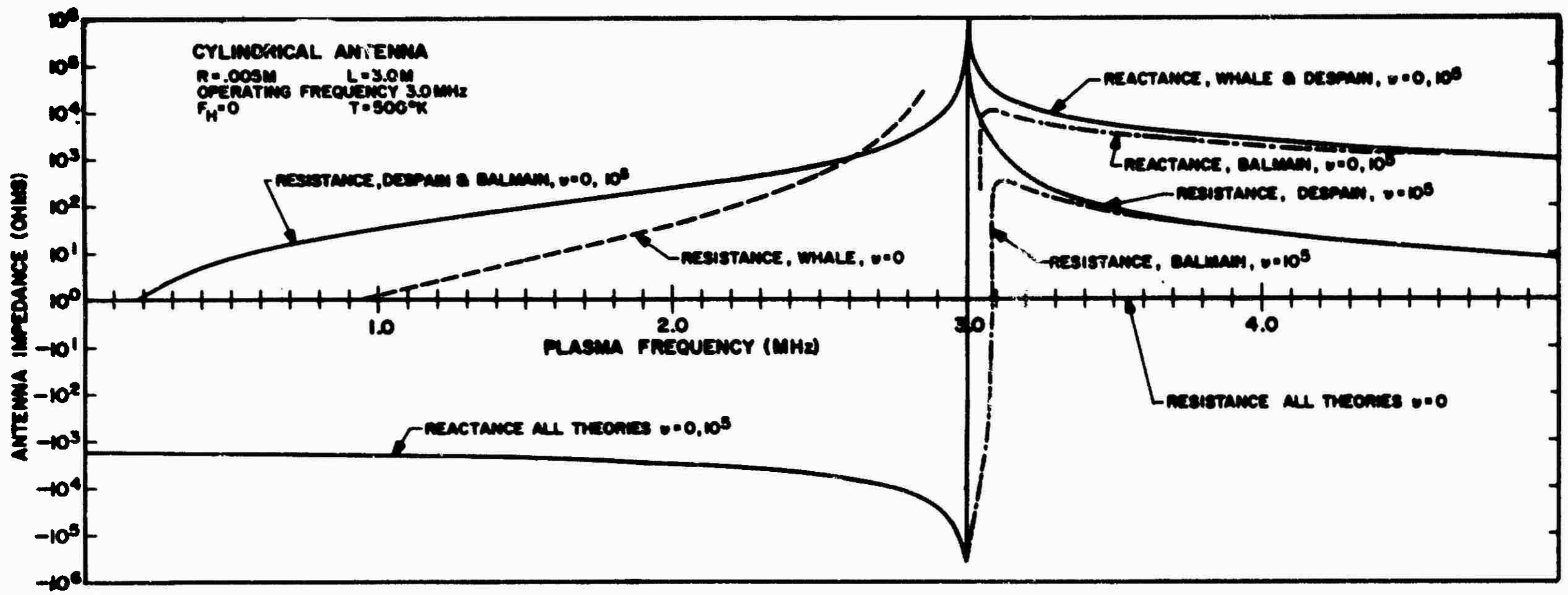


Fig. 11. Comparison of Despain, Balmain, and Whale Theories.

is negative for high electron temperatures in certain regions. This is clearly an impossible result for the physical system; and as discussed above, comparisons in these regions are not possible. Again, it appears that the Despain and Balmain theories generally agree over those regions where valid comparisons can be made.

Although only a few sets of parameters have been examined here; more extensive comparisons of the two theories have been made for several other sets of parameters typical of ionospheric experiments. In each case, the conclusions are identical to those discussed above.

In addition to the results from the theories of Balmain and Despain, the impedance expression as derived from Whale's theory is also presented in Figure 11. It can be noted that all the theories predict finite antenna resistance for finite electron temperature over at least part of the frequency range. Also the resistance increases with increasing plasma frequency in each case. The similarity ends at this point, however, since the Whale theory resistance curve is clearly in variance with both the Balmain and Despain theory. It is felt that because both the Despain and Balmain theories were more rigorously derived than Whale's work, that they are the more accurate formulations of antenna impedance. Therefore, Whale's work will not be considered further.

The Despain theory has been compared to other theoretical results. It was found to agree with Balmain's work where comparisons were possible. It did not agree with the Whale theory. Thus the Despain theory seems to be at least as good theoretically as the Balmain theory and yet it was derived from a somewhat different viewpoint and is somewhat more general in its formulation.

Important Implications of the Theory

There are several very important implications that can be noted in the examples of the Despain theory given in Figures 10 and 11. First, for no electron collisions and zero electron temperature, no antenna resistance appears. However, if finite electron temperatures exist, then antenna resistance appears for exciting frequencies above the plasma frequency. Thus one effect of electron temperature is to add a loss mechanism not considered in the usual cold plasma formulations. Second, it can be observed that for exciting frequencies below the plasma frequency, a series (low impedance) resonance exists, whose frequency is a sensitive function of the electron temperature. Above the plasma frequency, the electron temperature has a much smaller effect on the antenna impedance. Third, the electron density (plasma frequency), temperature, and collision frequency all greatly affect antenna impedance. Hence, it should be possible to directly determine those parameters from antenna impedance measurements. This problem is the subject of the next chapter.

Further Examination of the Theory

The effect of magnetic field has not yet been investigated. Therefore, in Figure 12, the gyrofrequency has been taken as 1.4 MHz. By comparing Figures 10 and 12, it can be seen that the primary effect of the magnetic field is to transform the frequency scale so that the zero reactance point of the zero temperature curve is transformed up to the gyrofrequency and the parallel resonance (high impedance) point is shifted from the plasma frequency to the upper hybrid frequency given by

$$f_P^2 = f_N^2 + f_H^2 \tag{103}$$

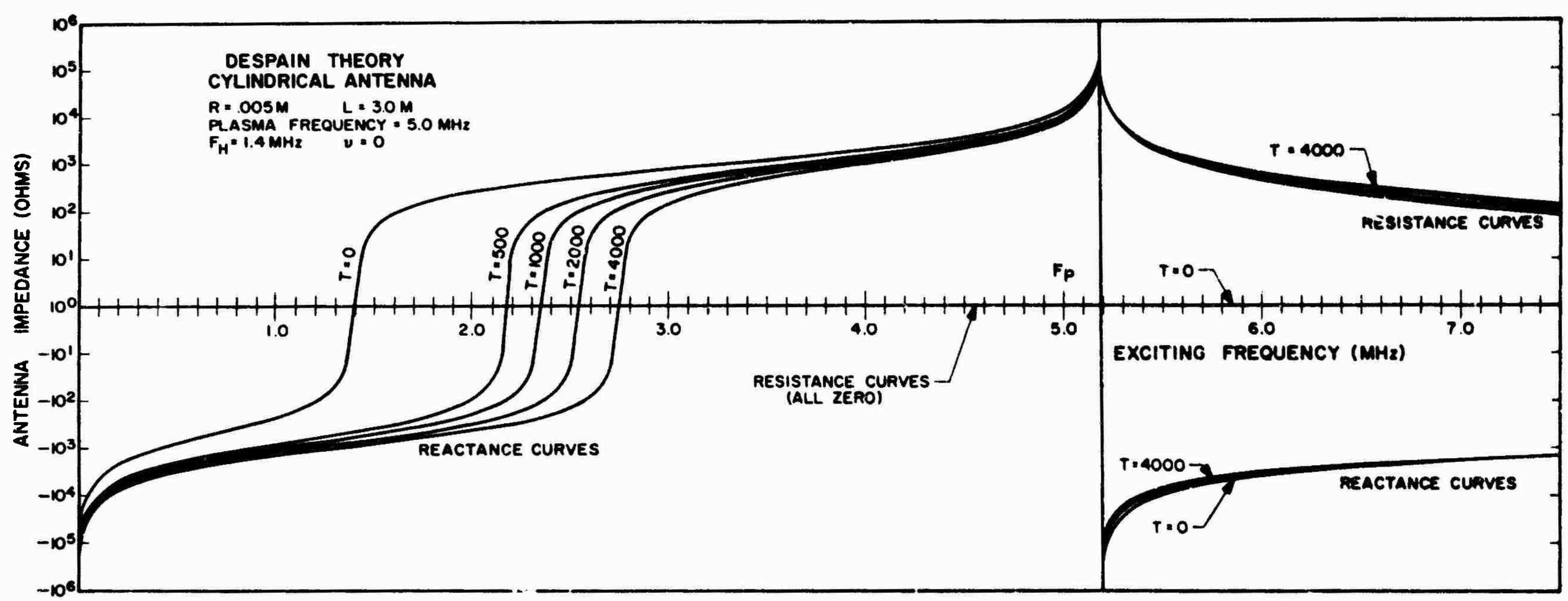


Fig. 12. Antenna impedance in the presence of a longitudinal magnetic field.

If a close examination is made, it is found that the impedance at f with no magnetic field is about the same at f' with longitudinal magnetic field where

$$f' = \sqrt{f^2 + f_H^2}$$

The additional effect of collisions is shown in Figure 13. It can be noted that the resonances are not changed in frequency although some change is evident in the shape of the reactance curves. Furthermore, large values of resistance are evident throughout the complete frequency range. Thus the primary result of electron collisions is an increase in antenna resistance.

## Comparison of the Theory to Experimental Results

There are two basic methods for comparing the theory to experimental results. One method is to use the theory to simulate antenna impedance from the best independent estimate of the experimental plasma parameters and to then compare these results with the experimental antenna impedance values. The other method is to use the theory to determine the plasma parameters from the experimentally determined antenna impedance values. The resulting plasma parameters are then compared to the independent estimate of the actual plasma parameters. The first technique will be employed in this section while the second will be used in the next chapter.

There are three crucial impedance characteristics predicted by the theory. The first is a parallel resonance condition for an exciting frequency near the upper hybrid frequency described by

$$f_P^2 = f_N^2 + f_H^2 \tag{105}$$

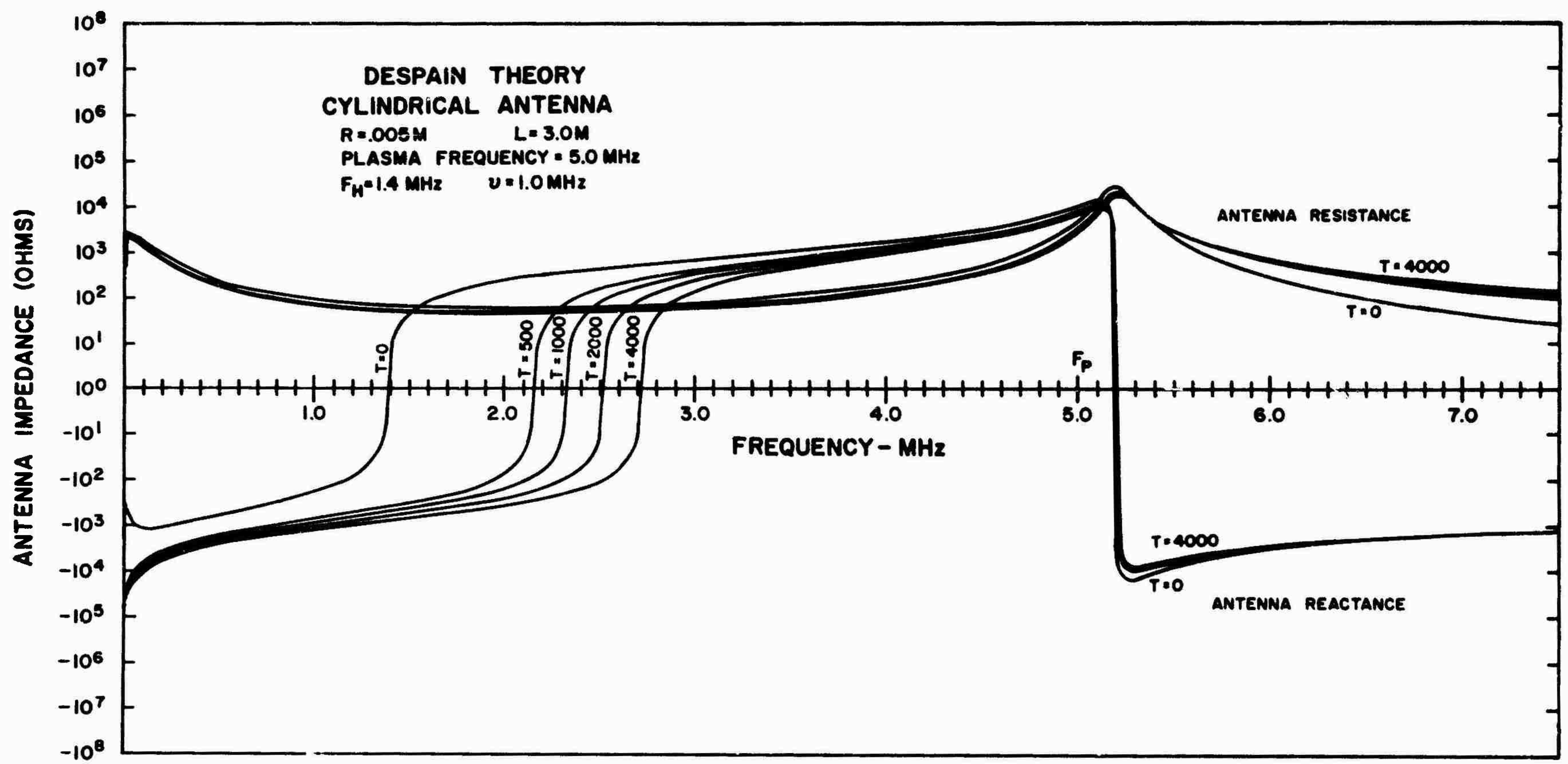


Fig. 13. Antenna impedance with longitudinal magnetic field and electron collisions.

The second is the existance of an electron temperature controlled series resonance condition for an exciting frequency between the gyrofrequency and the upper hybrid frequency. The third is the appearance of appreciable antenna losses for exciting frequencies above the upper hybrid frequency even when the electron collision losses are neglegible. Thus each of these characteristics should be observed in experimental results whenever the proper conditions are met.

Experimentally, series and parallel resonances have been observed many times in results from the Plasma Frequency Probe. In fact, whenever it could be established that according to the theory, that these resonances should occur, they were indeed observed. In most cases, independent measurements of plasma frequency were also made so that the parallel resonance condition could be checked. It was indeed at the predicted hybrid frequency in every case. Several such comparisons are discussed in a paper by *Baker, Despain and Ulwick* [1966] Independent measurements of electron temperature were not made in most cases. Thus even though the series resonances were consistently observed in all cases, it is not possible to determine at exactly what frequency they should have occurred. A typical example from one of these experiments is shown in Figure 14. Each experimental measurement is shown as a dot. A curve has been drawn through the mean of these points as a representation of the parallel frequency. There was evidence for this flight, that vehicle potential was rapidly changing over a several volt range. Thus it has not been possible to completely determine the antenna potential except to note that it varied reppeatedly from about zero down to about minus five volts during a period of about 1.6 seconds. Sheath effects that correspond to a value of S from about zero to four must therefore be effective here. This accounts for some of the spread in the series resonance data. So that this data could be compared to theory, a pair of curves were

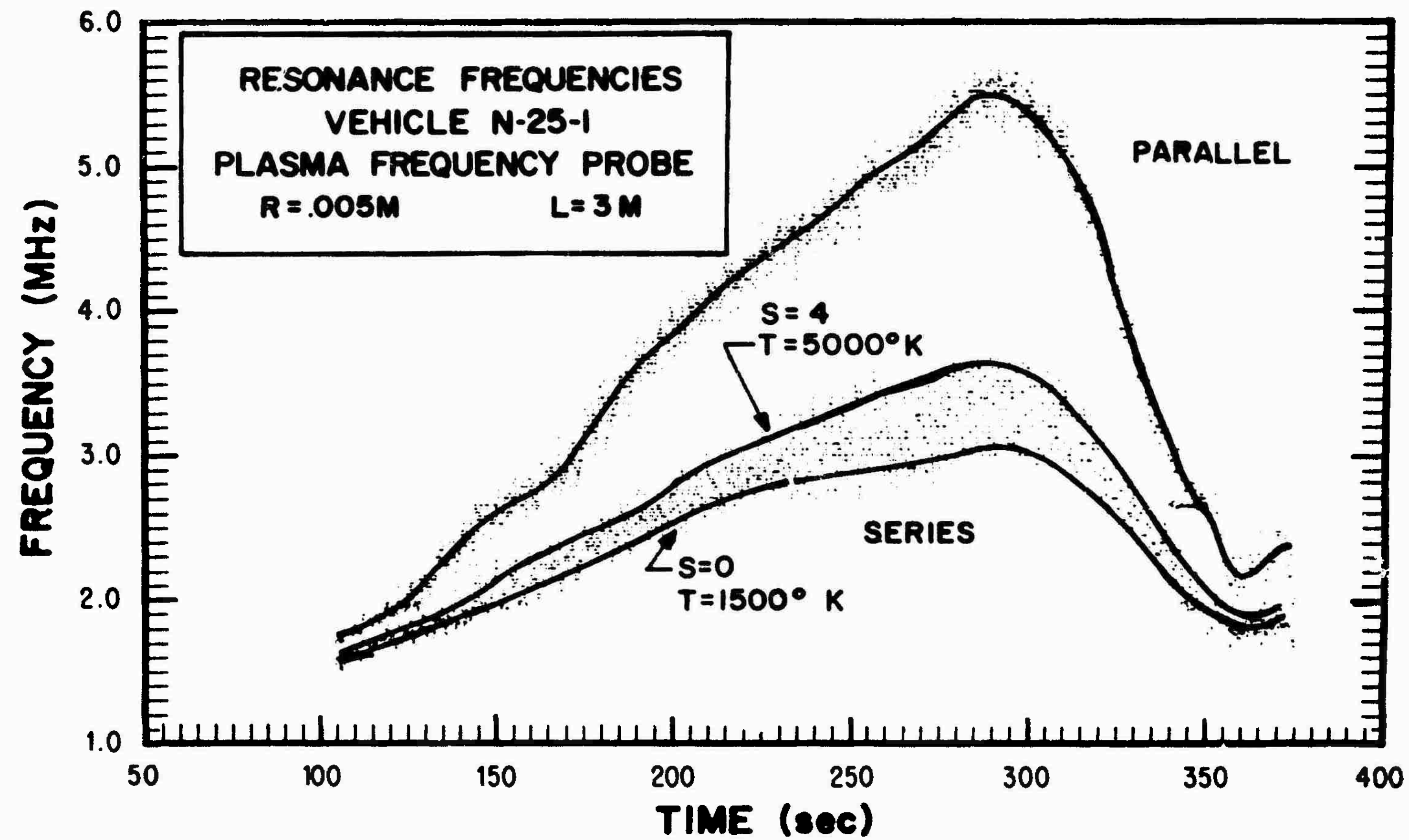


Fig. 14. Experimental series and parallel resonance frequencies

drawn near the series resonance points. These curves represent the predictions of the theory using the parallel resonance curve and the indicated sheath and electron temperature parameters. These parameters are respectively, the highest and lowest limits that can be reasonably estimated for this flight for the sheath variable S and electron temperature T. It can be seen that in this particular case, and within the experimental uncertainties, there is good agreement between theory and experiment. This same conclusion was reached after an examination of the other flights where good measurements were obtained.

The observation of high antenna losses in regions of low electron collision frequency have been observed by *Whale*[1964] as mentioned earlier. A recent experiment using the Standing Wave Impedance Probe also exhibited a similar phenomena [*Baker et al.*, 1965]. Since impedance measurements of electron temperature are not available, it is again impossible to predict exactly what losses should have been observed.

It is possible to assert that at least these experiments demonstrate results consistent with the theory and no contradictions are yet evident Such is not the case for the various other theories discussed earlier. The cold plasma theories and Whale's results do not predict a series resonance,and Balmain's warm plasma theory ignores magnetic fields that are always present in the ionosphere. Thus Balmain's theory does not predict correct values of series resonance or antenna losses. It is not possible to assert that the Despain theory is absolutely correct, but it has been proved that it is more accurate for the range of parameters of interest here than have any of the other theories of which the author is aware.

## Chapter V

## DETERMINATION OF PLASMA PARAMETERS

### Introduction

Historically, RF antenna measurements have been used to determine electron density (plasma frequency) and electron collision frequency, using one of the magneto-ionic theories in the interpretation of the data. From the theory developed in this paper, it can be seen that electron temperature must also be considered; indeed, it is possible to determine electron temperature from the antenna impedance measurements using this theory. The purpose of this chapter is, therefore, to demonstrate how electron density, temperature and collision frequency can all be determined by use of this theory.

Antenna impedance is described by two parameters; resistance and reactance. It is desired, however, to determine three parameters; that is, electron density, temperature and collision frequency. Thus either impedance measurements must be made at (a) more than one exciting frequency or (b) simultaneously on at least two antennas of significantly different configuration or (c) there must be some known relation between at least two of the plasma parameters.

Over the usual range of natural ionosphere conditions, there is a simple relationship between electron collision frequency, electron temperature and total particle density [*Pfister*, 1965]. The relation is

$$\nu = C_N T N_t \tag{106}$$

where $N_t$ = Total particle density (particles/meter$^3$)

$C_N$ = A constant derived from any desired model atmosphere, taken to be $2.6 \times 10^{-17}$ in this paper.

The total density can be found from a model atmosphere once the desired altitude is specified. The altitude, in turn, is known from the rocket trajectory. Thus the parameters altitude and antenna impedance can specify electron density, temperature, and collision frequency.

## Plasma Parameters from the Standing Wave Impedance Probe

The Standing Wave Impedance Probe determines antenna impedance at a single frequency and at a given time. Thus for a given altitude, a plot of antenna impedance as a function of plasma frequency and electron temperature can be used to determine these parameters. Such a plot is generated by a computer program known as IMPPLT in conjunction with the subroutine ANTZ. These programs are listed in Appendix B for reference.

A plot of antenna resistance versus reactance is shown in Figure 15 for $\nu = 0$, and for no magnetic field. It describes a typical antenna used in ionospheric experiments of 3 meter length, 1/2 cm radius and driven at 3 MHz. The impedance values shown are the differential impedance changes from the free space conditions. Curves are shown for various constant electron temperatures and also for various constant plasma frequencies. Figure 16 is a similar plot for $\nu = 1$ MHz in which the effect of electron collisions can be discerned. The primary effect of the collisions is to increase the antenna resistance. In Figure 17, the collisions are assumed to follow the relation given in equation (106) at 75 km. In addition, magnetic field effects have also been taken into account by assuming a gyrofrequency of 1.4 MHz. It is evident that given antenna impedance and altitude, the plasma frequency and electron temperature can be determined directly, and that equation (106) can specify electron collision frequency. Thus these parameters can all be determined from the results of the Standing Wave Impedance Probe.

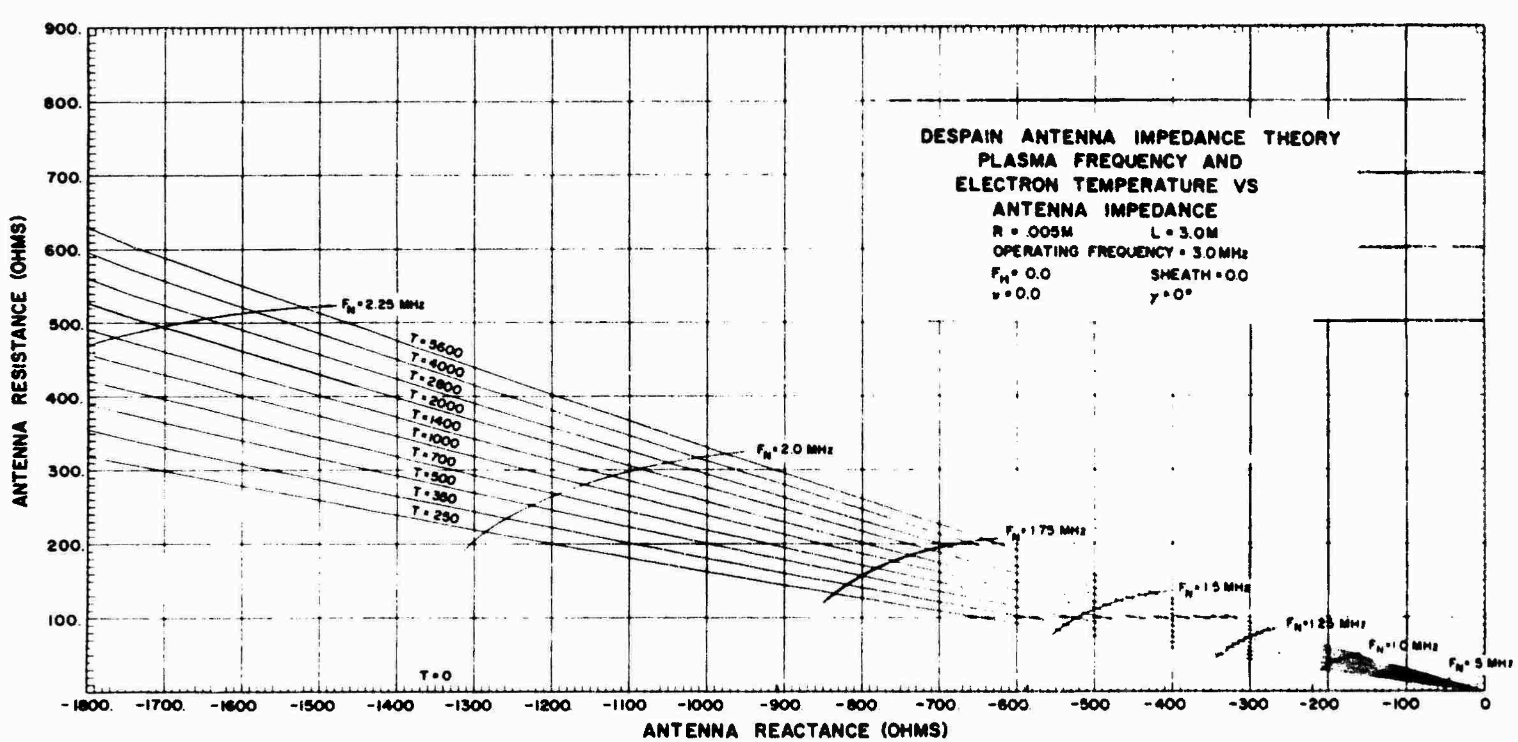


Fig. 15. Antenna resistance versus reactance.

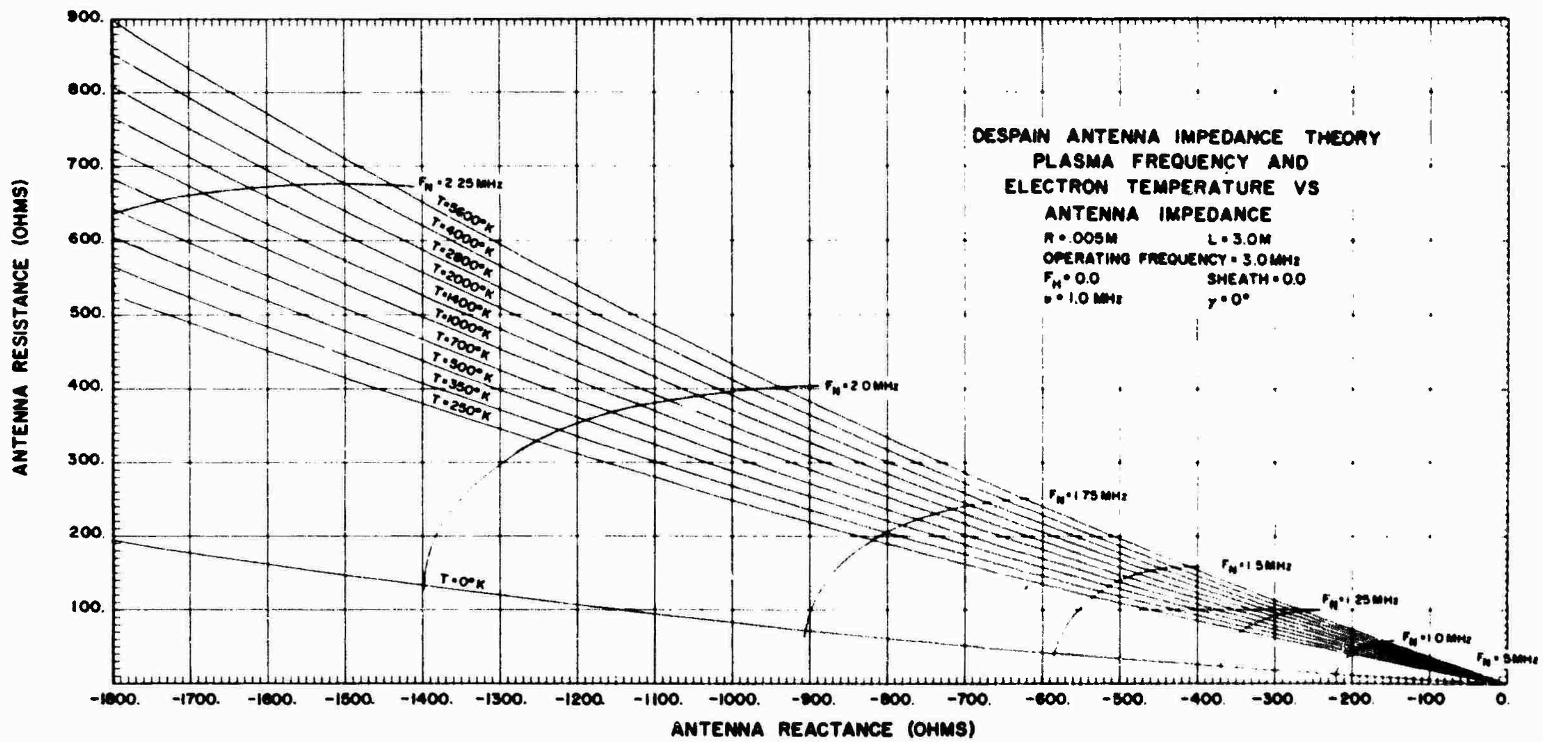


Fig. 16. Antenna resistance versus reactance including electron collisions.

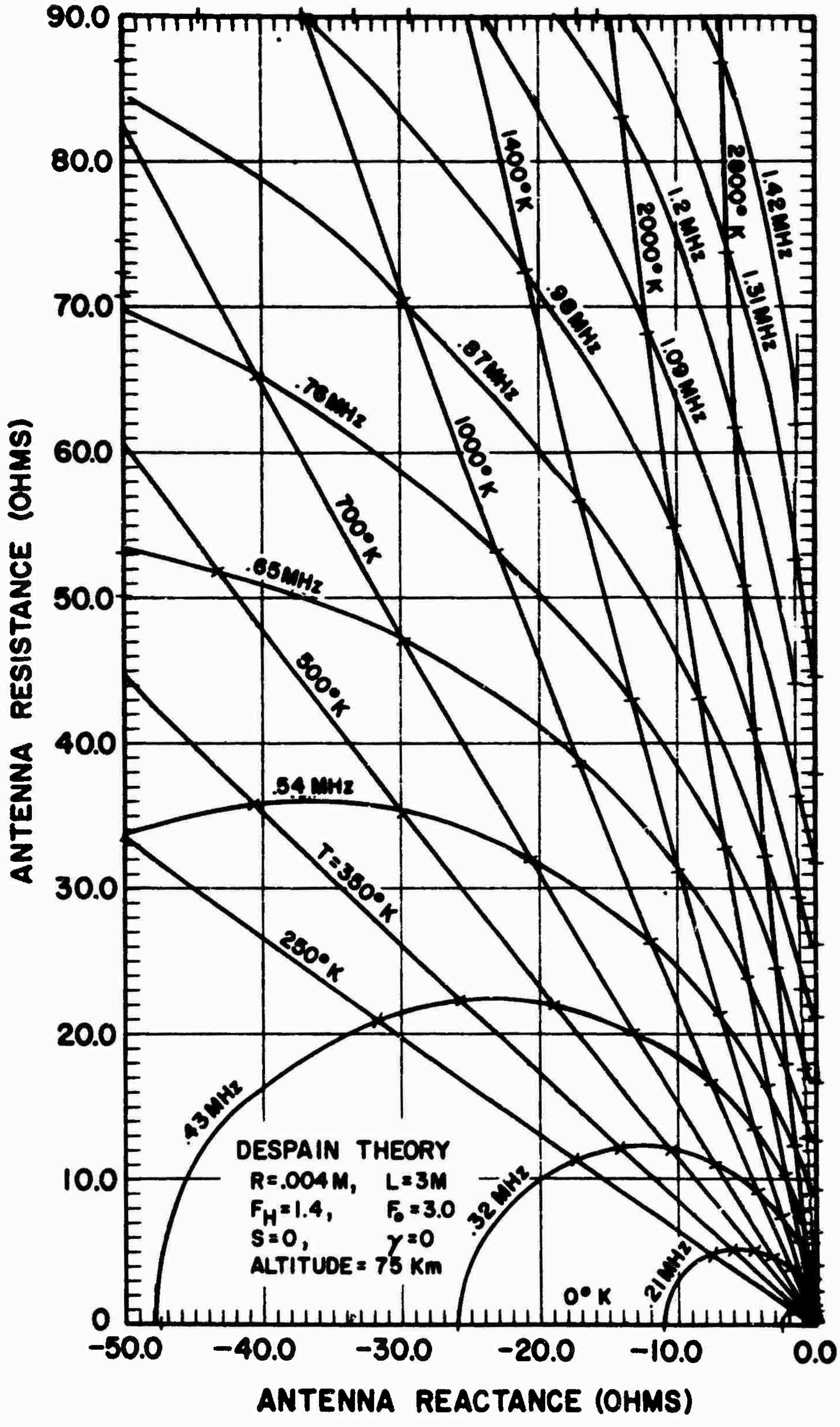


Fig. 17. Antenna Resistance versus reactance for 75 km.

## Plasma Parameters from the Plasma Frequency Probe

The Plasma Frequency Probe, as explained in Chapter I, directly determines the series and parallel resonance frequencies of an antenna that occur in the range of .1 MHz to 10 MHz. It was shown in the last chapter that the resonance frequencies are not modified by modest electron collision frequencies so that $\nu$ is assumed to be zero in the work that follows. The parallel frequency is not a function of electron temperature, and hence it defines the plasma frequency. The relationship for longitudinal magnetic field is

$$f_N^2 = f_P^2 - f_H^2 \tag{107}$$

For other magnetic aspect angles, it can be easily shown from magneto-ionic theory (Appendix A) that this equation produces at most an error of about 10% in plasma frequency. For $f_N \gg f_H$, the error is very small. The series frequency, on the other hand, is a function of plasma frequency, gyrofrequency and electron temperature. Thus it is desirable to calculate the series resonance frequency as a function of the antenna and plasma parameters so that this relationship can be examined.

## Calculation of Series Resonance Frequency

The condition for series resonance is that the imaginary part of the antenna impedance be equal to zero. Thus from Table I

$$\mathit{Imag}\left[\frac{1}{j\omega C_s} + \frac{1}{j\omega C_o}\left(\frac{\phi(\alpha R_s)/\ln(L/R_s) - \psi^2}{1-\psi^2}\right)\right] = 0 \tag{108}$$

or

$$C_o/C_s + \mathit{Real}\left[\frac{\phi(\alpha R_s)/\ln(L/R_s) - \psi^2}{1-\psi^2}\right] = 0 \tag{109}$$

If $\psi^2$ is found, the series frequency will be defined. Thus since $\nu = 0$, $\phi$ and $\psi$ are both real for

$$f_H < f < f_P \tag{110}$$

then

$$\psi^2 = [\ln(R_s/R_A) + \phi(\alpha R_s)]/\ln(L/R_A) \tag{111}$$

where

$$C_o/C_s = \ln(R_s/R_A)/\ln(L/R_s) \tag{112}$$

Equation (111) is the series respnance condition. It is a transcendental equation and is therefore not subject to analytical solution. It can be solved by a numerical technique such as Newton's iterative procedure. Subroutine SICALC of Appendix B solves the equation for $\psi$ by Newton's method. The series frequency $f_s$ is then given by the definition of $\psi$ as

$$f_s = [\psi^2 f_N^2 - f_H^2]^{1/2} \tag{113}$$

A computer program named TMPC1 (see Appendix B) calculates and plots this series frequency by calling SICALC and several other routines. The result is a plot of series versus parallel frequency with electron temperature as a parameter for a given set of plasma and antenna parameters. Figure 18 is an illustration of such a graph for a typical antenna. It can be seen from the figure that given the series and parallel resonance frequencies, the electron temperature can be determined.

The parameters of antenna radius, magnetic field and sheath radius all have some influence on the series resonance frequency. To illustrate the effect of these parameters, the curves of Figure 19 were developed. Note the decrease of series frequency as the antenna radius is increased

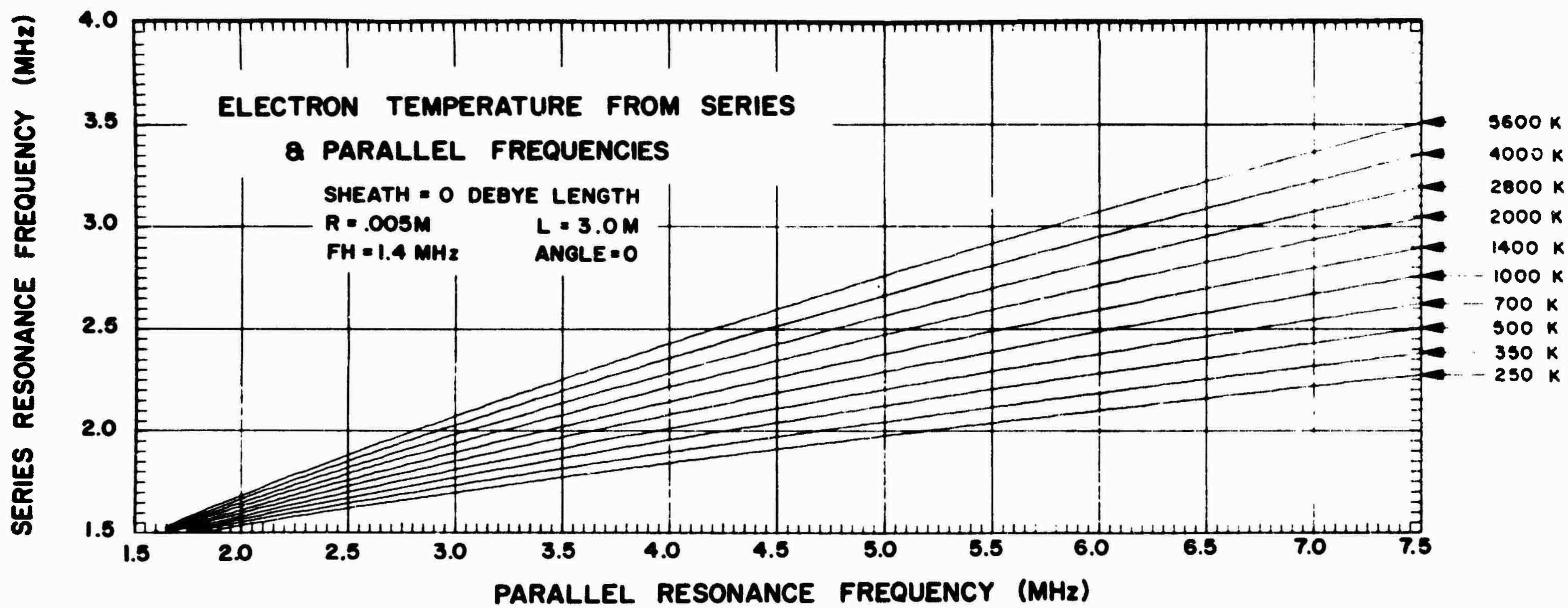


Fig. 18. Series versus parallel resonance frequencies.

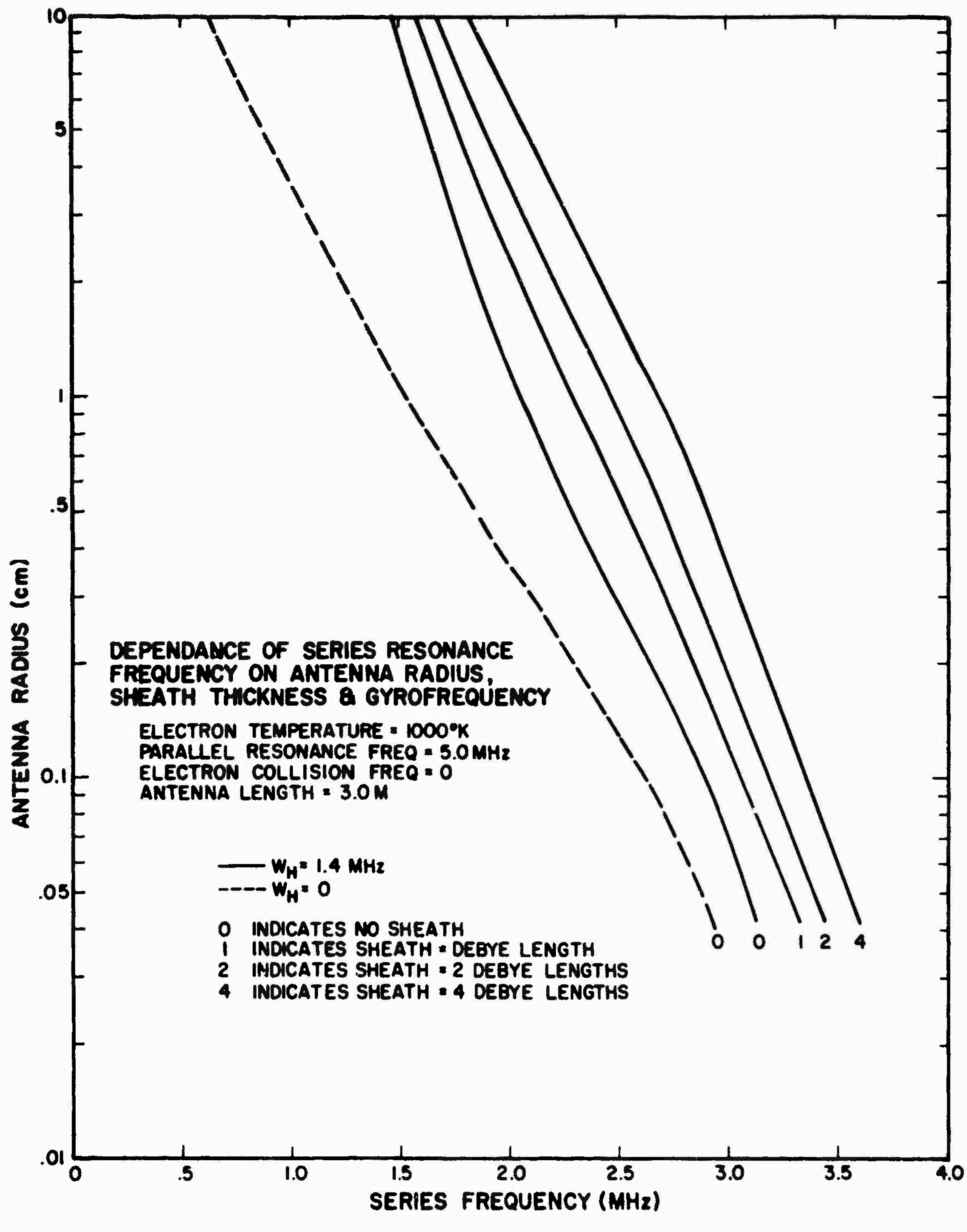


Fig. 19. Antenna radius versus series resonance frequency

and the increase in series frequency as magnetic field and sheath width are increased. Thus a knowledge of the sheath and magnetic field are required in addition to the antenna parameters and resonance frequencies in order to define electron temperature.

Electron Temperature from Resonance Frequencies

Electron temperature can be found from diagrams similar to Figure 19, or they can also be calculated directly. If electron temperature is desired, the resonance condition (109) is used to define $\Phi$ as follows

$$\Phi(\alpha R_s) = [\psi^2 + (1-\psi^2)C_o/C_s] \ln(L/R_s) \tag{114}$$

where all the parameters on the right hand side are known. Now $\Phi$ is a transcendental function of $\alpha R_s$ and while $\alpha R_s$ cannot be expressed analytically, it can be found numerically from the value of $\Phi$. Subroutine ETAA determines $\alpha R_s$ from the value of $\Phi$ by a series of simple iterations. The parameter $\alpha$ is a function of the electron temperature T and thus

$$T = (4\pi^2 R_s^2 m/3K)\ (f_s^2 - f_H^2)\ (1-\psi^2)/(\alpha R_s)^2 \tag{115}$$

This temperature is calculated by a subroutine named TEMPSP. The routines ETAA and TEMPSP are used in the main line program TEP601 to find electron temperature from the series and parallel resonance frequencies. Again the programs are listed in Appendix B.

Example of Electron Temperatures from Measured Antenna Characteristics

To illustrate the measurement of electron temperature, the results from a Plasma Frequency Probe flown on an Aerobee rocket AC 3.603 were analyzed. This rocket was fired into an active aurora; and hence, some large variations of the plasma parameters occurred, including high electron temperatures. The complete experiment has been described in detail

elsewhere [*Ulwick, et al.*, 1965] and an extensive analysis of the RF probe experiments aboard has also been reported [*Baker et al.*, 1965]. The important feature of this particular flight was the independent measurement of electron temperature by two other instruments: a planer retarding potential analyzer and an RF electron temperature probe (RF perturbated Langmuir probe, part of the Resonance Rectification Probe).

Sheath variations occurred with this experiment. However, the vehicle and antenna potential could be defined, and it was found that the antenna potential was zero at certain times during the flight [*Richards*, 1965]. The experimentally determined potentials are shown in Figure 20 as a function of the voltage step applied between the vehicle and the antenna. It can be seen that at step 5, the antenna potential is near zero and that the sheath must be collapsed for this condition. Thus the theory for no sheath can be applied at this time to determine electron temperature.

Program TEP601 was used to calculate electron temperature for the step 5 case over a limited region of the flight where good results were obtained. The results are shown in Figure 21, along with measurements from the other two instruments. There is considerable scatter in the measurement of the resonance frequencies due to telemetry noise and other experimental limitations. Hence the temperature values show large variations from what must be the true values. In addition, the true temperature values must also vary greatly as well, since the antenna is in an active auroral region characterized by disturbed temperature and density variations. Since each independent instrument measures at an independent time, some differences between instruments is expected due solely to the variations in the ionospheric medium. The general agreement between all the measurements is seen to be quite good when the above arguments are kept in mind.

In an attempt to further observe the correlation between the various

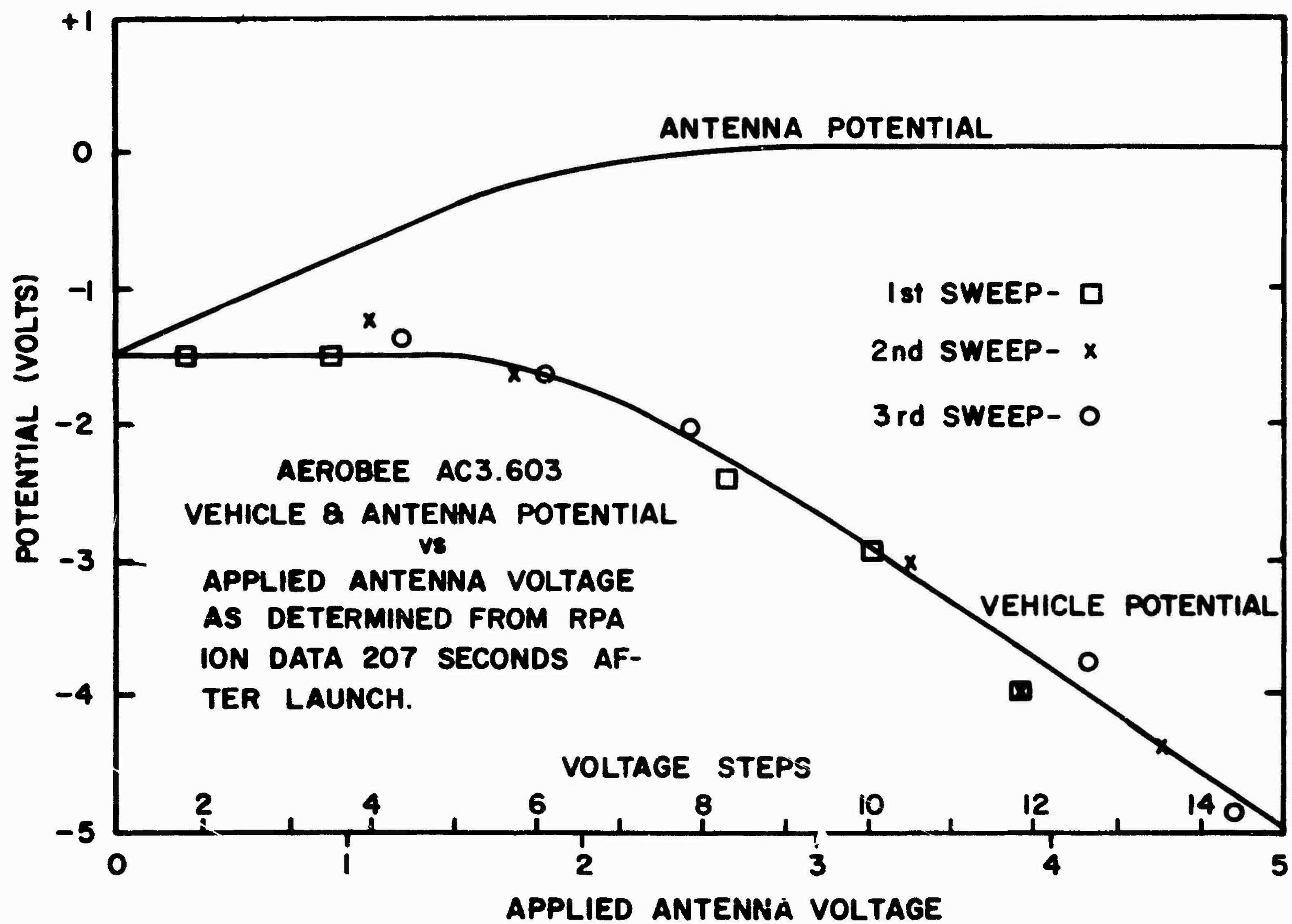


Fig. 20. Antenna and vehicle potential.

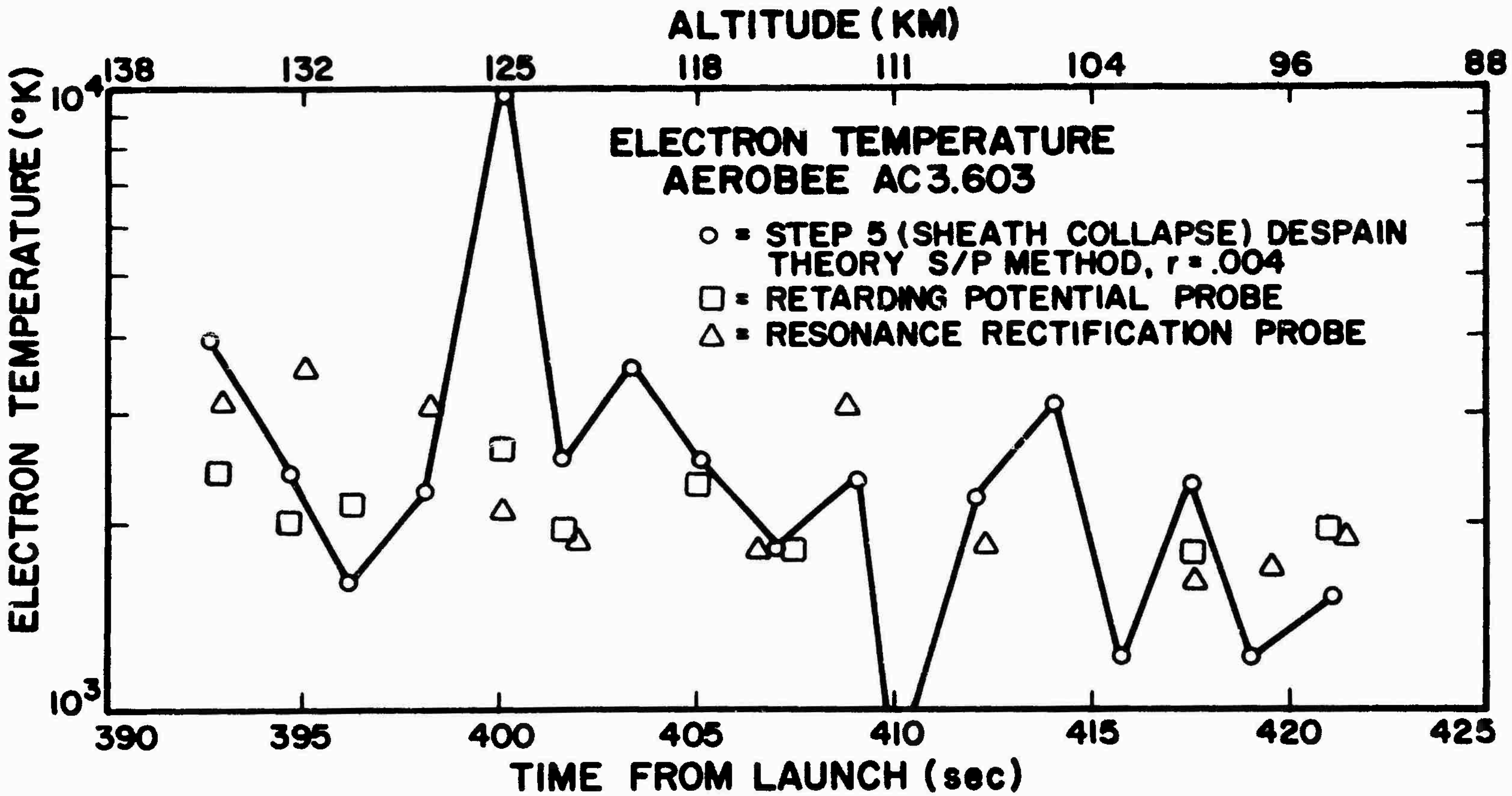


Fig. 21 Collapsed sheath electron temperatures.

techniques, the data under all sheath conditions was analyzed with the collapsed sheath theory and the results were averaged. This average curve is compared to the other measurements in Figure 22. Again reasonable correlations were obtained. It should be noted that the experiment was not designed to determine electron temperature and that further experimental refinements with this goal in mind would greatly improve the quality of the data. In general, it is felt that the measurements of electron temperatures by the resonance frequencies could be a very accurate technique, especially for experimental designs optimized for accurate determination of the resonance frequencies.

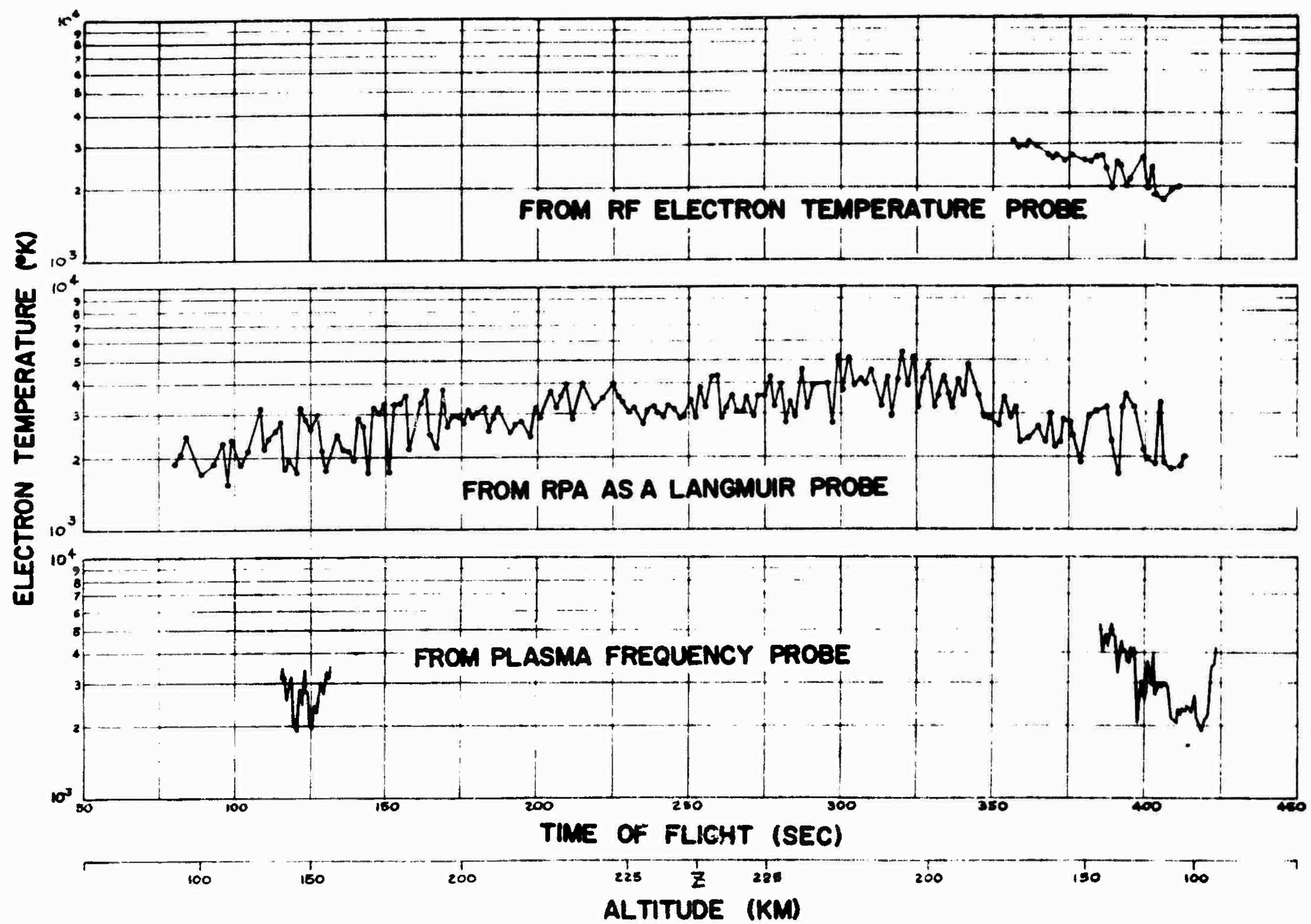


Fig. 22. Averaged electron temperatures - AC 3.603.

## Chapter VI

## SUMMARY AND CONCLUSIONS

This paper first discussed the plasma state and the important parameters needed to specify the character of a plasma. Then the range of parameters encountered in the ionospheric plasma was examined. Various RF probe techniques were presented and a need was shown for a theory to describe the plasma - antenna interaction so that measurements from these probes could be interpreted correctly.

Various theories have been previously presented but while some of these were suitable for a restricted range of plasma parameters, none of these theories could account for all the experimental results as observed in data from the probe experiments. Thus clearly further theoretical work was required.

In the theoretical work, the antenna was assumed to be cylindrical, electrically short, yet with sufficient length as compared to the radius so that end effects could be ignored. The surrounding plasma was characterized as a homogeneous, compressible electron fluid surrounding relatively fixed positive ions so that macro charge neutrality is maintained. The plasma properties are described by the plasma frequency, gyrofrequency and the electron temperature and collision frequency.

The theoretical approach considered the forces on an element of the electron fluid. In addition to the usual acceleration term, there were forces due to electric and magnetic fields, electron collisions and gradients of pressure in the electron fluid. The pressure gradients were related to density gradients and electron temperature through the ideal gas law and equation of state. The velocity of the electron fluid was found from this

force equation. This expression was then combined with the continuity and Poisson's equation to produce a second order partial differential equation in terms of the electron density variations. Solutions to this equation were found to be expressible as the well known Hankel functions. Boundary conditions then determined a unique solution. From a knowledge of the charge density, Poisson's equation was solved and the potential in the region was then defined. The fields were then easily determined and antenna impedance was found as the ratio of antenna potential to antenna current as derived from the field quantities. The usual discontinuous sheath model was employed to account for sheath effects. The result was an impedance expression that included all the important plasma parameters discussed earlier, especially electron temperature.

The results of the theoretical calcuation were numerically simulated with a computer, and the emperical results were compared to several other theories. The theory exhibited good agreement with magneto-ionic theories and also with Balmain's work for a range of parameters in which it was valid to make comparisons. In addition, a comparison was made to experimental results. The agreement was good, at least to within the limits of the accuracy of the experimental parameters. In contrast to all the other theories, no contradictions were noted between the theory and experimental results. Thus the theory appears, at least, to be the best available for interpretation of ionospheric RF probe experiments.

The problem of determining plasma parameters from impedance measurements was considered next. First it was demonstrated that plasma frequency, electron temperature, and collision frequency could be obtained from the altitude and a single frequency impedance measurement. Next it was shown how plasma frequency and electron temprature are defined by the series and parallel resonances of the antenna. Finally an example of determining electron

temperature from experimental Plasma Frequency Probe results was considered. The electron temperatures determined with the aid of the theory were compared to two other independent measurements and good correlations were obtained.

It is believed that the experimental electron temperatures derived by use of the theory are the first reported, direct electron temperature measurements of the ionosphere that do not use some variation of the Langmuir Probe Technique (see Chapter I). The theory therefore makes possible a valuable new method of measuring electron temperature.

It should be noted that the theory was developed to interpret RF probe results. Hence, the antennas were always very short compared to the RF wavelengths; and therefore, the electromagnetic effects were dominated by the plasma effects. In addition, the free space impedance of the antenna systems could be experimentally determined; and thus, the theory was only required to account for changes in the impedance due to the plasma.

In conclusion, it can be noted that the theory successfully accounts for all the experimental phenomena observed. It agrees well with other theories where comparisons are valid. Finally, its use makes possible, for the first time, the determination of electron temperature by an RF impedance probe technique.

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# APPENDIXES

APPENDIX A

ANTENNA IMPEDANCE FROM IONIC THEORY

## Isotropic Media-Capacitor Model

If magnetic fields and thermal motions other than collisions are ignored, then antenna impedance can be simply determined from ionic theory.

The frequencies of interest are in the range .1 to 10 MHz. Thus the shortest wavelength involved is 30 meters. In almost all cases, the antenna probes under consideration will be much shorter than this, typically about 3 meters. Therefore, the antennas are always short compared with the operating wavelengths, and hence may be considered as capacitors whose dielectric reflects the properties of the plasma medium that surrounds the probes. Capacitance of an object is defined as the ratio of the total charge stored on the surface to the electric potential of the object.

$$C = Q/\Phi \tag{A1}$$

In terms of the electric flux parameter $\vec{D}$, the capacitance is from Gauss's Law

$$C = \frac{\iint_S \vec{D}\cdot d\vec{S}}{\Phi} \tag{A2}$$

In terms of the free space capacitance $C_o$ and from the relation valid in free space $\vec{D} = \varepsilon_o\vec{E}$, it is found that

$$\frac{C}{C_o} = \frac{\iint_S \vec{D}\cdot d\vec{S}}{\varepsilon_o \iint_S \vec{E}\cdot d\vec{S}} \tag{A3}$$

For the well behaved geometries encountered in practice (planes, cylinders, etc.), it is permissible to consider both D and E invariant in magnitude and perpendicular with respect to the surface. Thus these quantities can be taken outside the integrals and the following relation results

$$\frac{C}{C_o} = \frac{\varepsilon}{\varepsilon_o} \tag{A4}$$

where

$$\vec{D} = \varepsilon\vec{E}$$

The antenna impedance $Z_a$ can now be obtained from

$$Z_a = 1/j\omega C$$

*Ratcliffe* [1962] derives an expression for $\varepsilon$ expressed by

$$\varepsilon = \varepsilon_o \left[1 - \frac{X}{1-jZ}\right] \tag{A5}$$

where

$X = \omega_N^2/\omega^2$

$Z = \nu/\omega$

$\omega_N$ = radian plasma frequency

$\omega$ = radian operating frequency

$\nu$ = electron collision frequency

Thus the antenna impedance $Z_a$ is

$$Z_a = \frac{1}{j\omega C} = \frac{1}{j\omega C_o} \left[1 - \frac{X}{1-jZ}\right]^{-1} \tag{A6}$$

or

$$Z_a = \frac{1}{\omega C_o}\left[\frac{XZ}{(1-X)^2 + Z^2} - j\,\frac{(1-X) + Z^2}{(1-X)^2 + Z^2}\right] \tag{A7}$$

In this expression, it is assumed that the antenna probe does not radiate. In actual practice however, even a very electrically short antenna will radiate some energy. This is often described by a radiation resistance $R_r$

$$R_r = P_r/I_a^2 \tag{A8}$$

where

$P_r$ = total radiated power

$I_a$ = antenna current at terminals.

*Kraus* [1950] in (5-55) gives the following expression for $R_r$ of a short dipole

$$2R_r = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{\beta^2(2L)}{6\pi} \tag{A9}$$

or

$$R_r = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{\beta^2 L}{6\pi} \tag{A10}$$

where

$R_r$ = resistance of monopole

$L$ = length of monopole

$\beta$ = $\omega/c$ = $2\pi/\lambda$

$c$ = velocity of light

$\lambda$ = wavelength

In a plasma medium $\mu_o$ is invariant, but as shown above, $\varepsilon$ varies according to (A5). Thus, the antenna impedance $Z_{ar}$ arising from the radiation can be determined by substituting (A5) into (A10). (K.D. Baker, private communication, 1964) has examined this problem and has verified that both the real and imaginary parts of the radiation resistance should be included in the total expression for antenna impedance. A similar approach for the more general case of antenna impedance has been developed by *Harrison and Denton* [1959], where they have let *Imag* [$\varepsilon$] be represented by $-\sigma/\omega$ where $\sigma$ is the conductivity of the medium.

The antenna resistance is thus

$$Z_{ar} = \frac{\sqrt{\mu_o}\beta^2 L}{6\pi}\left[1 - \frac{X}{1-jZ}\right]^{-1/2} \tag{A11}$$

and the total impedance $Z_{at}$ is then

$$Z_{at} = \frac{1}{j\omega C_o}\left[1 - \frac{X}{1-jZ}\right]^{-1} + \frac{\sqrt{\mu_o}\beta^2 L}{6\pi}\left[1 - \frac{X}{1-jZ}\right]^{-1/2} \tag{A12}$$

The radiation term should be included whenever the antenna is not extremely short or when very small changes of resistance are of interest.

Magneto-Ionic Cold Plasma

If a steady magnetic field permeates the plasma, the medium is no longer isotropic. Thus while (A3) still applies, the relation between $\vec{D}$ and $\vec{E}$ involves a tensor [$\varepsilon$]. Since E is always normal to the antenna surface, (A3) can be expressed as

$$\frac{C}{C_o} = \frac{\iint_s \frac{\vec{D}\cdot\vec{E}}{|E_s|}\, dS}{\varepsilon_o \iint_s |E_s|\, dS} \tag{A13}$$

where $|E_s|$ = magnitude of E at the antenna surface.
But by definition of $[\varepsilon]$

$$\vec{D}\cdot\vec{E} = \sum_{i,j} E_i \varepsilon_{ij} E_j \tag{A14}$$

or

$$\vec{D}\cdot\vec{E} = E_x^2\,\varepsilon_{11} + E_y^2\,\varepsilon_{22} + E_z^2\,\varepsilon_{33} + \sum_{\substack{i,j\\ i\neq j}} t_{ij} \tag{A15}$$

where $t_{ij}$ = all cross products.

Thus (A13) becomes

$$\frac{C}{C_o} = \frac{\iint_s \frac{E_x^2\varepsilon_{11}}{|E_s|}\,dS + \iint_s \frac{E_y^2\varepsilon_{22}}{|E_s|}\,dS + \iint_s \frac{E_z^2\varepsilon_{33}}{|E_s|}\,dS + \iint_s \frac{t_{ij}}{|E_s|}\,dS}{\varepsilon_o \iint_s |E_s|\,dS} \tag{A16}$$

At the surface of the cylindrical antenna

$$E_x = |E_s| \cos\theta \tag{A17}$$

$$E_y = |E_s| \sin\theta \tag{A18}$$

$$E_z = 0 \tag{A19}$$

Thus since $E_z = 0$ and the $t_{ij}$ integrals all go to zero as they contain cross terms that integrate to zero, the equation becomes

$$\frac{C}{C_o} = \frac{\varepsilon_{11}}{\varepsilon_o}\,\frac{\int_o^L\int_o^{2\pi} |E_s| \cos^2\theta\,d\theta\,dr}{\int_o^L\int_o^{2\pi} |E_s|\,d\theta\,dr} + \frac{\varepsilon_{22}}{\varepsilon_o}\,\frac{\int_o^L\int_o^{2\pi} |E_s| \sin^2\theta\,d\theta\,dr}{\int_o^L\int_o^{2\pi} |E_s|\,d\theta\,dr} \tag{A20}$$

It is assumed that $E_s$ is not a function of θ. Thus (A20) becomes

$$\frac{C}{C_o} = \frac{\varepsilon_{11}}{\varepsilon_o} \frac{1}{2\pi} \int_o^{2\pi} \mathrm{Cos}^2 \theta d\theta + \frac{\varepsilon_{22}}{\varepsilon_o} \frac{1}{2\pi} \int_o^{2\pi} \mathrm{Sin}^2 \theta d\theta \tag{A21}$$

$$\frac{C}{C_o} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2\varepsilon_o} \tag{A22}$$

According to Appleton-Hartree Theory [*Ratcliffe,* 1962]

$$\varepsilon_{11} = \varepsilon_o \left[1 + \frac{XU^2}{U(Y^2 - U^2)}\right] \tag{A23}$$

$$\varepsilon_{22} = \varepsilon_o \left[1 + \frac{X(U^2 - Y^2 \mathrm{Sin}^2 \gamma)}{U(Y^2 - U^2)}\right] \tag{A24}$$

Thus

$$\frac{C}{C_o} = 1 + 1/2 \left[\frac{XU^2 + X(U^2 - Y^2 \mathrm{Sin}^2 \gamma)}{U(Y^2 - U^2)}\right] \tag{A25}$$

$$\frac{C}{C_o} = 1 + X \frac{1 - 1/2\ (Y/U)^2 \mathrm{Sin}^2 \gamma}{U(Y/U)^2 - 1} \tag{A26}$$

or antenna impedance is

$$Z_a = \frac{1}{j\omega C} = \frac{1}{j\omega C_o} \left[1 + X \frac{1 - 1/2\ (Y/U)^2 \mathrm{Sin}^2 \gamma}{U(Y/U)^2 - 1}\right]^{-1} \tag{A27}$$

This expression is the expression for antenna impedance from the magneto-ionic theory of a cold plasma.

Comparison to Balmain Theory

The antenna impedance of (A27) can be directly compared to the results of Balmain (see Equation 10). As mentioned in the main text of this report, Balmain's work is probably the best published to date on cold plasma antenna impedance. To compare these two theories, they were written up as Fortran subroutines known as ANTZ - BALMAIN and ANTZ-MAGNETO-IONIC THEORY. These routines were successively used in a program called SWP4MD that calculated and plotted the antenna impedance as a function of plasma frequency and for fixed exciting frequency (see Appendix B). The results have shown very good empirical agreement except when the exciting frequency is near the plasma frequency. A typical sample of both theories is shown in Figure A-1 and illustrates the areas of disagreement. *Stone, Weber and Alexander* [1966] have also noted a theoretical discrepancy between Balmain's theory and the theory of Herman and others when the plasma frequency is in the vicinity of the exciting frequency. In addition, it appears that their experimental evidence confirms that Balmain's results are not valid in this region. Thus it may well be that the magneto-ionic theory presented here is more correct and much easier to derive than the theory of Balmain. It should also be noted here that this derivation is similar in many respects to the derivation of *Crouse* [1964] made on a rocket nose cone and thus should agree very well with his results if the cone angle is set to zero.

Hot Plasma

The basic assumptions and view of the plasma in this section are identical to those developed in Chapter III. The strategy of deriving the antenna impedance differs in that it is assumed that the plasma can be represented by an effective permittivity tensor $[\epsilon]$. Thus the same basic

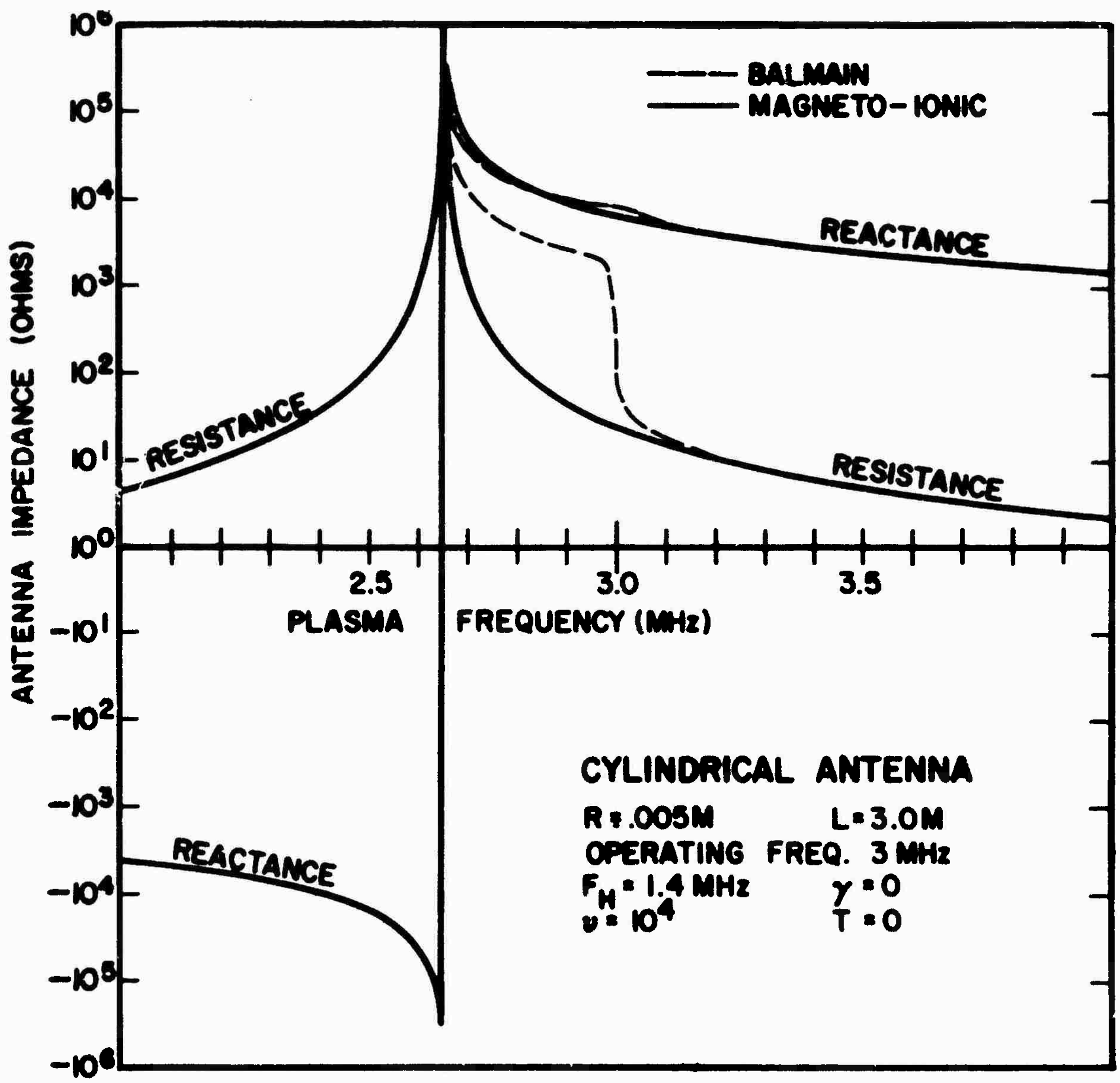


Fig. A-1. Comparison of Balmain and magneto-ionic antenna theories.

equations are used to find [ε], but instead of developing partial differential equations as in Chapter III, this derivation derives the antenna impedance directly from ε by an integration process similar to that used above for the cold plasma case.

Permittivity [ε]

The starting point in deriving [ε] is the force equation (25).

$$j\omega N\,[U\vec{v}-jY(\vec{v}\times\hat{\zeta})] = -\mu^2\nabla n + \frac{Ne}{m}\nabla\Phi \tag{A28}$$

Where

$$\vec{E} = -\nabla\Phi \tag{A29}$$

$$\nabla^2\Phi = \frac{ne}{\varepsilon_o} \tag{A30}$$

the force equation becomes

$$j\omega N\,[U\vec{v}-jY(\vec{v}\times\vec{\zeta})] = \frac{\mu^2\varepsilon_o}{e}\nabla(\nabla\cdot E) - \frac{Ne}{m}\vec{E} \tag{A31}$$

Now the usual assumption is made [*Ratcliffe*, 1962, p. 15] that the velocity can be represented by a polarization P as follows

$$\vec{P} = -Ne\vec{r} \tag{A32}$$

$$\dot{\vec{P}} = +j\omega\vec{P} = -Ne\vec{v} \tag{A33}$$

Thus

$$\vec{v} = -\frac{j\omega}{Ne}\vec{P} \tag{A34}$$

and the force equation becomes

$$\varepsilon_o X\vec{E} - \frac{\mu^2 \varepsilon_o}{\omega^2} \nabla(\nabla \cdot \vec{E}) = -[U\vec{P} - jY\vec{P} \times \hat{\zeta}] \tag{A35}$$

where

$$X = \omega_N^2/\omega^2 = Ne^2/m\varepsilon_o\omega^2 \tag{A36}$$

Now tensor notation is introduced.

Let

$$[\Delta] = \begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial x\partial z} \\ \frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial y\partial z} \\ \frac{\partial^2}{\partial x\partial z} & \frac{\partial^2}{\partial y\partial z} & \frac{\partial^2}{\partial z^2} \end{bmatrix} \tag{A37}$$

and

$$[\sigma]^{-1} = \begin{bmatrix} -U & -jY_z & jY_y \\ jY_z & -U & -jY_x \\ -jY_y & -jY_x & -U \end{bmatrix} \tag{A38}$$

where

$$\vec{Y} = Y\hat{\zeta} = Y_x\hat{x} + Y_y\hat{y} + Y_z\hat{z} \tag{A39}$$

Then the force equation can be written as

$$\left[\varepsilon_o X - \frac{\mu^2 \varepsilon_o}{\omega^2} [\Delta]\right] \vec{E} = [\sigma]^{-1}\vec{P} \tag{A40}$$

However

$$\vec{D} = [\varepsilon]\vec{E} = \varepsilon_o\vec{E} + \vec{P} \tag{A41}$$

or

$$\vec{P} = [\varepsilon]\vec{E} - \varepsilon_o[I]\vec{E} \tag{A42}$$

Thus

$$\left[\varepsilon_o X - \frac{\mu^2\varepsilon_o}{\omega^2}[\Delta]\right]\vec{E} = [\sigma]^{-1}\left[[\varepsilon] - \varepsilon_o[I]\right]\vec{E} \tag{A43}$$

$$\varepsilon_o X[\sigma] - \frac{\mu^2\varepsilon_o}{\omega^2}[\sigma]\,[\Delta] = [\varepsilon] - \varepsilon_o\,[I] \tag{A44}$$

or

$$[\varepsilon] = \varepsilon_o\left[X[\sigma] + [I] - \frac{\mu^2}{\omega^2}[\sigma]\,[\Delta]\right] \tag{A45}$$

now

$$X[\sigma] = -\left[\frac{X}{U(U^2-Y^2)}\right]\begin{bmatrix}(U^2-Y_x^2) & (-jUY_z-Y_xY_y) & (jUY_y-Y_xY_z)\\ (jUY_z-Y_yY_x) & (U^2-Y_y^2) & (-jUY_x-Y_yY_z)\\ (-jUY_y-Y_zY_x) & (jUY_x-Y_zY_y) & (U^2-Y_z^2)\end{bmatrix} \tag{A46}$$

[*See Ratcliffe*, 1962; p. 182-184]

$$[I] = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \tag{A47}$$

Let

$$-\frac{\mu^2}{\omega^2}[\sigma]\,[\Delta] = \frac{VX}{U(U^2-Y^2)}[A] \tag{A48}$$

where

$$V = \mu^2/\omega_N^2$$

then the elements of [A] are

$$A_{11} = (U^2-Y_x^2)\frac{\partial^2}{\partial x^2} - (jUY_z-Y_xY_y)\frac{\partial^2}{\partial x\partial y} + (jUY_y-Y_xY_z)\frac{\partial^2}{\partial x\partial z} \quad \text{(A49)}$$

$$A_{12} = (U^2-Y_x^2)\frac{\partial^2}{\partial x\partial y} - (jUY_z-Y_xY_y)\frac{\partial^2}{\partial y^2} + (jUY_y-Y_xY_z)\frac{\partial^2}{\partial y\partial z} \quad \text{(A50)}$$

$$A_{13} = (U^2-Y_x^2)\frac{\partial^2}{\partial x\partial z} - (jUY_z-Y_xY_y)\frac{\partial^2}{\partial y\partial z} + (jUY_y-Y_xY_z)\frac{\partial^2}{\partial z^2} \quad \text{(A51)}$$

$$A_{21} = (jUY_z-Y_xY_y)\frac{\partial^2}{\partial x^2} + (U^2-Y_y^2)\frac{\partial^2}{\partial x\partial y} - (jUY_x-Y_yY_z)\frac{\partial^2}{\partial x\partial z} \quad \text{(A52)}$$

$$A_{22} = (jUY_z-Y_xY_y)\frac{\partial^2}{\partial x\partial y} + (U^2-Y_y^2)\frac{\partial^2}{\partial y^2} - (jUY_x-Y_yY_z)\frac{\partial^2}{\partial y\partial z} \quad \text{(A53)}$$

$$A_{23} = (jUY_z-Y_xY_y)\frac{\partial^2}{\partial x\partial z} + (U^2-Y_y^2)\frac{\partial^2}{\partial y\partial z} - (jUY_x-Y_yY_z)\frac{\partial^2}{\partial z^2} \quad \text{(A54)}$$

$$A_{31} = -(jUY_y-Y_xY_y)\frac{\partial^2}{\partial x^2} + (jUY_x-Y_yY_z)\frac{\partial^2}{\partial x\partial y} + (U^2-Y_z^2)\frac{\partial^2}{\partial x\partial z} \quad \text{(A55)}$$

$$A_{32} = -(jUY_y-Y_xY_z)\frac{\partial^2}{\partial x\partial y} + (jUY_x-Y_yY_z)\frac{\partial^2}{\partial y^2} + (U^2-Y_z^2)\frac{\partial^2}{\partial y\partial z} \quad \text{(A56)}$$

$$A_{33} = -(jUY_y-Y_xY_z)\frac{\partial^2}{\partial x\partial z} + (jUY_x-Y_yY_z)\frac{\partial^2}{\partial y\partial z} + (U^2-Y_z^2)\frac{\partial^2}{\partial z^2} \quad \text{(A57)}$$

Now let $[\varepsilon] = \varepsilon_o \dfrac{-X}{U(U^2-Y^2)}[T]$ (A58)

Then the elements of [T] are given by

$$\varepsilon_{ij} = \varepsilon_o\left[X\sigma_{ij} + I_{ij} + \frac{VX}{U(U^2-Y^2)}A_{ij}\right] = \varepsilon_o\frac{-X}{U(U^2-Y^2)}T_{ij} \quad \text{(A59)}$$

$$T_{ij} = -U(U^2-Y^2)\delta_{ij} - (U/X)\ (U^2-Y^2)I_{ij} - VA_{ij} \tag{A60}$$

Thus

$$T_{11} = (U^2-Y_x^2) - (U/X)(U^2-Y^2) - VA_{11} \tag{A61}$$

$$T_{12} = -(jUY_z-Y_xY_y) - VA_{12} \tag{A62}$$

$$T_{13} = (jUY_y-Y_xY_z) - VA_{13} \tag{A63}$$

$$T_{21} = (jUY_z-Y_xY_y) - VA_{21} \tag{A64}$$

$$T_{22} = (U^2-Y_y^2) - (U/X)(U^2-Y^2) - VA_{22} \tag{A65}$$

$$T_{23} = -(jUY_x-Y_yY_z) - VA_{23} \tag{A66}$$

$$T_{31} = -(jUY_y-Y_xY_z) - VA_{31} \tag{A67}$$

$$T_{32} = (jUY_x-Y_yY_z) - VA_{32} \tag{A68}$$

$$T_{33} = (U^2-Y_z^2) - (U/X)(U^2-Y^2) - VA_{33} \tag{A69}$$

Therefore $[\varepsilon]$ is

$$[\varepsilon] = \varepsilon_o \frac{-X}{U(U^2-Y^2)}\ [T] \tag{A70}$$

where the elements of $[T]$ are given above.

The permittivity tensor represents the hot, magneto-ionic medium. It should be noted that it contains partial derivatives with respect to the coordinates. Thus the spatial form of the $\vec{E}$ field must be specified before $[\varepsilon]$ is completely defined. This implies that $[\varepsilon]$ is a function of the coordinates.

## Hot Plasma Antenna Impedance

The E field around the cylindrical antenna is given by

$$\vec{E} = E(r)[\mathrm{Cos}\ (\varsigma)\ \hat{x} + \mathrm{Sin}\ (\epsilon)\hat{y}] \tag{A71}$$

and the magnetic field vector is assumed to lie in the y-z plane, at an angle $\gamma$ with respect to the z axis (see Figure 8 in the main text).

Thus

$$Y_x = 0,\ Y_y = Y\ \mathrm{Sin}\ \gamma,\ Y_z = Y\ \mathrm{Cos}\ \gamma \tag{A72}$$

$$E_x = E_r \mathrm{Cos}\ \theta \tag{A73}$$

$$E_y = E_r \mathrm{Sin}\ \theta \tag{A74}$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_x}{\partial r}\frac{dr}{dx} + \frac{\partial E_x}{\partial \theta}\frac{d\theta}{dx} \tag{A75}$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_r}{\partial r} + \frac{E_r}{r}\mathrm{Sin}^2\ \theta \tag{A76}$$

$$\frac{\partial^2 E_x}{\partial x^2} = \frac{2E_o}{r^3}\mathrm{Cos}^3\ \theta \tag{A77}$$

where

$$E_r = \frac{E_o}{r}$$

Similarly

$$\frac{\partial^2 E_x}{\partial x \partial y} = \frac{2E_o}{r^3}\ \frac{\mathrm{Cos}^2\theta(1+\mathrm{Sin}^2\theta)}{\mathrm{Sin}\ \theta} \tag{A78}$$

$$\frac{\partial^2 E_y}{\partial y^2} = \frac{2E_o}{r^3} \sin^3\theta \tag{A79}$$

$$\frac{\partial^2 E_y}{\partial x \partial y} = \frac{2E_o}{r^3} \frac{\sin^2\theta\,(1+\cos^2\theta)}{\cos\theta} \tag{A80}$$

All partial derivatives with respect to z are zero as are all partial derivatives of $E_z$.

Thus

$$\frac{1}{E_x}\frac{\partial^2 E_x}{\partial x^2} = \frac{\cos^2\theta}{r^2} \tag{A81}$$

$$\frac{1}{E_x}\frac{\partial^2 E_x}{\partial x \partial y} = \frac{2\cos\theta(1+\sin^2\theta)}{r^2 \sin\theta} \tag{A82}$$

$$\frac{1}{E_y}\frac{\partial^2 E_y}{\partial x \partial y} = \frac{2\sin\theta(1+\cos^2\theta)}{r^2 \cos\theta} \tag{A83}$$

$$\frac{1}{E_y}\frac{\partial^2 E_y}{\partial y^2} = \frac{2\sin^2\theta}{r^2} \tag{A84}$$

and

$$A_{11} = U^2 \frac{2\cos^2\theta}{r^2} - (jUY\cos\gamma)\frac{2\cos\theta(1+\sin^2\theta)}{r^2\sin\theta} \tag{A85}$$

$$A_{12} = U^2\frac{2\sin\theta(1+\cos^2\theta)}{r^2\cos\theta} - (jUY\cos\gamma)\frac{2\sin^2\theta}{r^2} \tag{A86}$$

$$A_{21} = jUY\cos\gamma\frac{2\cos^2\theta}{r^2} + (U^2-Y^2\sin^2\gamma)\frac{2\cos\theta(1+\sin^2\theta)}{r^2\sin\theta} \tag{A87}$$

$$A_{22} = jUY\cos\gamma\frac{2\sin\theta\,(1+\cos^2\theta)}{r^2\cos\theta} + (U^2-Y^2\sin^2\gamma)\frac{2\sin^2\theta}{r^2} \tag{A88}$$

It follows that

$$T_{11} = U^2-(U/X)(U^2-Y^2)-VU^2\,\frac{2\mathrm{Cos}^2\theta}{r^2} + jVUY\mathrm{Cos}\gamma\,\frac{2\mathrm{Cos}\,\theta(1+\mathrm{Sin}^2\theta)}{r^2\mathrm{Sin}\,\theta} \quad \text{(A89)}$$

$$T_{12} = -jUY\mathrm{Cos}\gamma-VU^2\,\frac{2\mathrm{Sin}\theta(1+\mathrm{Cos}^2\theta)}{r^2\mathrm{Cos}\,\theta} + jVUY\mathrm{Cos}\gamma\,\frac{2\mathrm{Sin}^2\theta}{r^2} \quad \text{(A90)}$$

$$T_{21} = jUY\mathrm{Cos}\gamma-V(U^2-Y^2\mathrm{Sin}^2\gamma)\,\frac{2\mathrm{Cos}\,\theta(1+\mathrm{Sin}^2\theta)}{r^2\mathrm{Sin}\theta} - jVUY\mathrm{Cos}\gamma\,\frac{2\mathrm{Cos}^2\theta}{r^2} \quad \text{(A91)}$$

$$T_{22} = U^2-Y^2\mathrm{Sin}^2\gamma-(U/X)(U^2-Y^2)-V(U^2-Y^2\mathrm{Sin}^2\gamma)\frac{2\mathrm{Sin}^2\theta}{r^2}$$

$$-jVUY\mathrm{Cos}\gamma\,\frac{2\mathrm{Sin}\,\theta(1+\mathrm{Cos}^2\theta)}{r^2\mathrm{Cos}\,\theta} \quad \text{(A92)}$$

Since

$$[\varepsilon] = \frac{-\varepsilon_o X}{U(U^2-Y^2)}\,[T] \quad \text{(A93)}$$

$[\varepsilon]$ is now defined. The antenna capacitance is given by (A13) and with (A14) it is possible to write

$$\frac{C}{C_o} = \frac{\dfrac{-\varepsilon_o X}{U(U^2-Y^2)}\displaystyle\iint_s \sum_{i,j} \frac{E_i T_{ij} E_j}{|E_s|}\,dS}{\varepsilon_o\displaystyle\iint_s |E_s|\,dS} \quad \text{(A94)}$$

now since $E_z = 0$, and $r = R_s$

$$\frac{C}{C_o} = \frac{-X}{U(U^2-Y^2)}\,\frac{1}{2\pi}\int_o^{2\pi} \frac{E_x T_{11} E_x}{|E_s|^2} + \frac{E_x T_{12} E_y}{|E_s|^2} + \frac{E_y T_{21} E_x}{|E_s|^2} + \frac{E_y T_{22} E_y}{|E_s|^2}\,d\theta \quad \text{(A95)}$$

by use of the following relations

$$\int_{o}^{\partial\pi} \mathrm{Sin}^2\,\theta d\theta = \pi \tag{A96}$$

$$\int_{o}^{\partial\pi} \mathrm{Cos}^2\,\theta d\theta = \pi \tag{A97}$$

$$\int_{o}^{\partial\pi} \mathrm{Sin}^4\,\theta d\theta = \frac{3\pi}{4} \tag{A98}$$

$$\int_{o}^{\partial\pi} \mathrm{Cos}^4\,\theta d\,\theta = \frac{3\pi}{4} \tag{A99}$$

$$\int_{o}^{\partial\pi} \mathrm{Sin}^2\theta\mathrm{Cos}^2\,\theta d\theta = \frac{2\pi}{8} \tag{A100}$$

and by noting that all the other trigonometric expression integrate to zero, it is found that

$$\frac{1}{2\pi}\int_{o}^{2\pi} \frac{E_x T_{11} E_x}{|E_s|^2}\, dS = \frac{1}{2}\left[U^2(1-3V/2R_s^2) - (U/X)(U^2-Y^2)\right] \tag{A101}$$

$$\frac{1}{2\pi}\int_{o}^{2\pi} \frac{E_x T_{12} E_y}{|E_s|^2} = -\frac{1}{2}\left[5U^2V/2R_s^2\right] \tag{A102}$$

$$\frac{1}{2\pi}\int_{o}^{2\pi} \frac{E_y T_{21} E_x}{|E_s|^2} = -\frac{1}{2}\left[(U^2-Y^2\mathrm{Sin}^4\gamma)(5V/2R_s^2)\right] \tag{A103}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{E_y{}^T 22\, E_y}{|E_s|^2} = \frac{1}{2}\,[U^2(1-3V/2R_s^2)-(U/X)(U^2-Y^2)] \tag{A104}$$

The capacitance is then

$$\frac{C}{C_o} = 1-(X/U)(1-(W^2/2)\mathrm{Sin}^2\gamma)(1-4V/R_s^2)/(1-W^2) \tag{A105}$$

This expression is identical to (A26) except for the $4V/R_s^2$ term that represents the effects of electron temperature. Thus (A105) is the antenna impedance from magneto-ionic theory including the effects of electron temperature.

## Simulated Results

The above expression (A105) was programmed and the results were plotted. An example is shown in Figure A-2. If it is compared to a similar curve from Despain Theory as developed in the main text, the comparison is very poor. In fact, the results shown are physically so unlike other results that they cannot be accepted even as an approximation. Thus this theoretical technique cannot be used when finite electron temperatures are under consideration.

The exact reason for the discrepancy is unknown to the author; however, it is his opinion that the use of a polarization vector P to represent the hot electrons is in error. This formulation does not allow for electroacoustic (longitudinal) waves to exist in the plasma as does the theory developed in the text. It may well be that for

$$4V/R_s^2 << 1$$

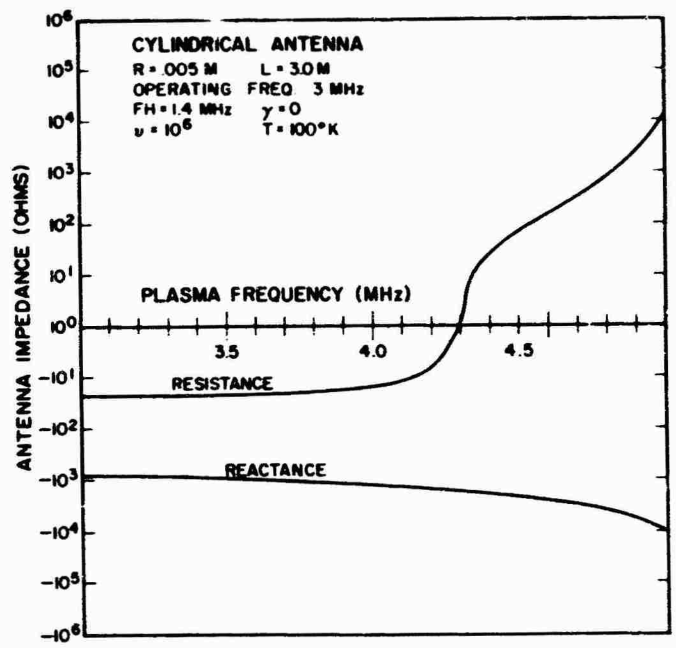


Fig. A-2. Magneto-ionic finite temperature theory impedance plot

that this formulation would produce correct results. This would be the case for either very high plasma frequencies or low electron temperatures. It is not suitable for the usual range of ionospheric parameters.

## APPENDIX B

## COMPUTER PROGRAMS

### Introduction

All of the following programs were coded in Fortran IV and processed on an IBM 7044 computer at the University of Utah Computer Center. The computer system includes a CalComp digital plotter. Extensive use was made of this device to display simulated and experimental results. There are many programs that are part of the IBM 7044 and CalComp system. These system subroutines will not be included here, but can be obtained from the University of Utah Computer Center. Only the special external routines developed for the work reported in this paper will be given.

There are four mainline programs and a number of subroutines, some of which are used in more than one of the main line programs. Hence if a given routine cannot be found after the main program in which it is called, then it will either be cataloged under one of the other main line programs or is a subroutine included in the 7044/CalComp system. Each subroutine called by a program can also call other subroutines and read input data cards. Thus before an attempt is made to use the programs, all subroutines should be checked for additional call statements. The Fortran IV program listings follow the descriptions of the routines.

### Main Program SWP4MD

SWP4MD is the main program for plotting theoretical antenna impedance as a function of frequency. The theory is contained in subroutine ANTZ of which there are four versions:

(a) Magneto-ionic theory
(b) Balmain theory
(c) Whale theory
(d) Despain theory

Any one of these can be used with the main program. The program produces a tabular listing of the frequency, impedance, and plot coordinates with a heading block on each page. It can also write a tape suitable for data storage and external plotting. It produces a CalComp plot of impedance versus frequency. Each plot is 8" x 15", the ordinate ranges from $-10^8$ to $+10^8$ ohms on a logarithmic scale. The abscissa begins at zero and ends at a specified maximum frequency (MHz). There are many options and input variables. These are explained below.

| Variable Name | Function |
|---|---|
| NPLØTS | Number of separate figures. |
| NRESLV | Number of separate data points per curve (resolution). |
| NTAPE | Identification of tape drive unit that output is written on. If negative or greater than 4, no tape is written. |
| NSKIP | Number of files skipped before data is written on tape unit NTAPE. |
| NPRINT | Ratio of total data points plotted and written on the tape to the data points printed on the listing. |
| NPLT | A branching constant. If set to zero, no plot is produced. If set to 1, a plot is produced including labels. If set equal to 2, the plot figure is labeled, but the curves are not. |
| MTHY | A branching constant such that if set to 2, the impedance is found as a function of plasma frequency, otherwise impedance as a function of operating frequency is determined. |
| NDØ | Number of curves per figure. |
| KGRID | A dummy variable. May be set to any value. |
| FMAX | Maximum desired frequency (MHz) on the abscissa of the figure. |

| Variable Name | Function |
|---|---|
| FX | The plasma frequency (MHz) is MTHY is 2, otherwise it is the operating frequency (MHz). |
| FH | Electron gyrofrequency (Hz). |
| FNU | Electron collision frequency (Hz). |
| T | Electron temperature (°K). |
| R | Antenna radius (m). |
| AL | Effective antenna length (m). |
| ANGLE | Angle between the antenna axis and the magnetic field vector (degrees). |
| RINIT | Resistance to be added to the antenna impedance and its shunt capacitance. |
| XINIT | Reactance divided by the applied frequency that is to be added to the antenna impedance if MODE is 1. Otherwise, it is the fixed inductance in series with the antenna and its shunt capacitance. |
| MØDE | A branching constant. If MØDE is set to 1, jXINIT*F and RINIT are added to the antenna impedance. If MØDE and MTHY are set to 2, the difference between free space and actual antenna impedance is produced. If MTHY is not 2 and MØDE is set to 2, the actual antenna impedance is produced. If MØDE is set to 3, first the shunt capacitance and then the series resistance and inductance are added to the antenna impedance. If set to 4, the difference impedance from free impedance is plotted, with first the shunt capacitance and then the series resistance and inductance taken into account. |

The following cards are called by the program:

| Card No. | Format | Variables |
|---|---|---|
| 1 | 7I10 | NPLØTS, NRESLV, NTAPE, NSKIP, NPRINT, MPLØT, MTHY |
| 2 | 13A6 | Major Title |
| 3 | 13A6 | Minor Title |
| 4 | 2I10,2F10.3, 3E 12.6,I4 | NDØ, KGRID, FMAX, FX, RINIT, XINIT, MODE |
| 5 | 6E13.6 | FH, FNU, T, R, AL, ANGLE |

There is one card similar to card 5 for every separate curve (the number of curves is given by NDØ). Thus, if NDØ should equal 3, cards 6 and 7 would be similar to card 5. Each separate figure requires the groups of cards that follow card 1. Thus, if NPLØTS should be 2, all the cards after card 1 would be repeated for the second figure.

The following subroutines are called by SWP4MD:

| Subroutine Name | Location | Function |
|---|---|---|
| CØNSTS | External | Calculate physical constants |
| SKPFLS | System | Skip over tape files |
| ANTZ | External | Calculate antenna impedance |
| IDPLØT | System | Generate plot identification |
| PLØT | System | Generate plot pen movements |
| SYMBL4 | System | Generate plot symbols |
| NUMBER | System | Generate plot numbers |
| FINI | System | Generate plot ending |

Subroutine ANTZ

There are several versions of ANTZ, each of which calculates antenna impedance according to several different theories. The primary variables are transferred through a labeled common statement as follows:

```
CØMMØN/IMPCØM/Z, F, FP, FNU, FH, ANGLE, T, R, AL
```

where

| Variable | Function |
|---|---|
| Z | Complex antenna impedance |
| F | Exciting frequency |
| FP | Plasma frequency |

| Variable | Function |
|---|---|
| FNU | Electron collision frequency |
| FH | Electron gyrofrequency |
| ANGLE | Magnetic aspect angle (degrees) |
| T | Electron temperature |
| R | Antenna radius (m) |
| AL | Antenna length (m) |

Physical constants are transferred through another labeled common statement /CØNSTA/. These constants can be obtained by a CALL CØNSTS statement in the main line program. Further definitions of the constants are included under the description of subroutine CØNSTS.

The Despain theory version of ANTZ also includes an additional labeled common statement /FPHIC/AR, PHI where

| Variable | Function |
|---|---|
| AR | Complex variable $\alpha R$ in the Despain theory |
| PHI | Complex variable $\phi$ in the Despain theory |

The Despain theory version of ANTZ calls subroutine APHICF that calculates PHI from AR and transfers the variables through the above described common statement. Subroutine APHICF must be included along with the main line and other programs when the Despain theory versions of ANTZ is used.

The Whale theory of ANTZ requires an interpolation subroutine INTERP, and this subroutine must be included when the Whale theory version of ANTZ is used.

Subroutine APHICF

This subroutine calculates the function PHI, given by

PHI = $H_o$ (AR)/(AR*$H_1$(AR))

where

$H_o$ = Hankel function of the first kind, zero order

$H_1$ = Hankel function of the first kind, first order

and PHI and AR are complex variables transferred through a labeled common statement as follows:

```
CØMMØN/APHIC/AR, PHI
```

Subroutine INTERP

INTERP is a interpolation subroutine that produces a interpolated value of YØUT for a given XIN from the arrays of data XA and YA where NPTS is the size of the array.

Subroutine CØNSTS

This subroutine calculates several physical constants. These constants are transferred through a labeled common statement as follows:

```
CØMMØN/CØNSTA/PI, PI2, EMASS, Q, CK, EO, G, UO, ALPHAK
```

The constants are as follows:

| Constants | Value |
|---|---|
| PI | 3.14159265 |
| PIZ | 2.0 * PI |
| EMASS | 9.1083E-31 |
| AMASS | EMASS*1836.12*26.5 |
| Q | 1.60206E-19 |
| CK | 1.38044E-23 |
| EO | 8.85434E-12 |

| Constants | Value |
|---|---|
| G | 3.0 |
| UO | PI*4.0E-07 |
| ALPHAK | 4.0*PI*PI*EMASS/(G*CK) |

Main Program TMPCI

TMPCI is the main program for calculating and plotting the series resonance frequency of an antenna versus the parallel resonance frequency with electron temperature as a parameter. The program calls the following subroutines in the given order:

| Subroutine Name | Location | Function |
|---|---|---|
| INPUT | External | Read in input data |
| GRID | External | Generate a plot grid |
| SICALC | External | Calculate $\psi$ function |
| CURVEL | External | Plot array of data as a line |
| ARRØW | System | Generate a plot arrow |
| PLØT | System | Generate plot pen movements |
| NUMBER | System | Write numbers on the plot |
| ØUTPUT | External | Generate output listing |
| FINI | System | End plotting |
| EXIT | System | End computing |

Subroutine INPUT (TMPCI)

INPUT reads in the input data for main program TMPCI. The following cards are called by this routine:

| Card No. | Format | Variables |
|---|---|---|
| 1 | 2I5,6E11.5 | NT, NFP, R, AL, FPMAX, FPMIN, SHEATH, FH |
| 2 | 5E16.8 | T(1), T(2), T(3), T(4), T(5) |
| 3 | 5E16.8 | T(6), T(7), T(8), T(9), T(10) |

where

| Variable Name | Function |
|---|---|
| NT | Number of curves per figure |
| NFP | Number of calculated points per curve |
| R | Antenna radius (m) |
| AL | Antenna length (m) |
| FPMAX | Maximum desired value of parallel frequency (MHz) |
| FPMIN | Minimum desired value of parallel frequency (MHz) |
| SHEATH | Desired size of sheath in Debye lengths |
| FH | Value of electron gyrofrequency |
| T | Electron temperature |

Subroutine SICALC

SICALC computes the $\psi$ function of the Despain antenna impedance theory. It is used in conjunction with main program TMPC1 and the input variables are described under the description of TMPC1. Newton's method is used in an iteration process to find $\psi$ from the imput parameters. The following subroutines are called by SICALC:

| Subroutine Name | Location | Function |
|---|---|---|
| CØNSTS | External | Calculation of constants |
| K | External | Calculation of Bessel Function |

Subroutine K

Subroutine K is a function subprogram. The call statement is K(N,X). It calculates the modified Bessel function of the second kind, of order N and argument X for N = 0,1. Subroutine I is called by subroutine K. X is a real variable.

Subroutine I

Subroutine I is a function subprogram. The call statement is I(N,X). It calculates the modified Bessel function of the first kind of order N and argument X for N = 0,1. X is a real variable.

Subroutine GRID

Subroutine GRID is a plot routine that draws a system of labeled coordinates 9" high and of variable length. There are four options controlled by TITLE, the first variable in the call list. Option 1 takes all the control constants from the call list. Option 2 reads the control constants from input cards and puts the values in the call list and in the labeled common statement /PLTCMN/. Option 3 reads the constants and puts them only in /PLTCMN/. Option 4 reads no cards but takes the variables from /PLTCMN/. The call list specifies the desired option, and the various possibilities are given below.

| Option | Call List |
|---|---|
| 1 | (TITLE, SCX, SCY, XXB, XXE, YYB, YYE, NXX, NYY, XX15) |
| 2 | (6HVREADR, SCX, SCY, XXB, XXE, YYB, YYE, NXX, NYY, XX15) |
| 3 | (6HbREADb) |
| 4 | (6HCØMMØN) |

The input variables are explained below:

| Variable Name | Function |
|---|---|
| TITLE and HEAD | These are dimentioned arrays of A format data. These data are the labels for the plot coordinates. |
| SCX and SCALEX | The number of major divisions along the abscissa. |
| SCY and SCALEY | The number of major divisions along the ordinate. |

| Variable Name | Function |
|---|---|
| XXB and XB | The smallest value of the variable to be plotted along the abscissa. |
| XXE and XE | The largest value of the variable to be plotted along the abscissa. |
| YYB and YB | The smallest value of the variable to be plotted along the ordinate. |
| YYE and YE | The largest value of the variable to be plotted along the ordinate. |
| NXX and NX | The number of significant figures in the abscissa number labels. |
| NYY and NY | The number of significant figures in the ordinate number labels. |
| XX15 and X15 | The desired length of the abscissa of the graph in inches. |

Cards are read where options 2 and 3 are used.

| Card No. | Format | Variables |
|---|---|---|
| 1 | 13A6 | HEAD (Label for abscissa) |
| 2 | 13A6 | HEAD (Label for ordinate) |
| 3 | 13A6 | HEAD (Major title of figure) |
| 4 | 13A6 | HEAD (Minor title of figure) |
| 5 | 6F10.5,2I5,F10.5 | SCALEX, SCALEY, XB, XE, YB YE, NX, NY, X15 |

No external subroutines are called. The result is a labeled graph X15" by 9", divided into SCALEX x SCALEX divisions with 10 minor divisions indicated along the edge of the figure.

Subroutine CURVEL

Curvel is a plot routine used in conjunction with subroutine GRID. Curvel generates a curve of line segments that connect the array of points

given by AX, AY, and N. The abscissa values are AX, the ordinate values are AY and N is the number of points or array values. AX and AY may be dimensioned at any desired value in the main program. The remaining arguments in the call list are scaling constants provided by subroutine GRID and have the same meaning as explained above.

Subroutine ØUTPUT (TMPC1)

ØUTPUT prints a listing of the results of subroutine SICALC in a tabular form with a heading block on each page. ØUTPUT is called by TMPTC1. No subroutines are called and no cards are read by ØUTPUT. All data is transferred through the labeled common statement /INPTS/.

Main Program TEP601

TEP601 is the main program for the calculation of electron temperature from the series and parallel resonance frequencies of the antenna impedance. TEP601 reads the input data, calls subroutine TEMPSP, and punches and prints both the input data and calculated temperature. The program variables are:

| Variable Name | Function |
|---|---|
| R | Antenna radius (m) |
| AL | Antenna length (m) |
| PERCNT | Allowable error in calculation of temperature (10% error would be put in as .10). |
| LIMIT | Maximum number of interations allowed in the recursive calculation of electron temperature. |
| CC | Value of any coupling capacitor (farads). If none exists, CC should equal 1.0. |
| S | Sheath width in number of Debye lengths. |

| Variable Name | Function |
|---|---|
| TSTART | Iteration starting value of electron temperature (typically 1000.0). |
| FMT | Format specification for main input data card deck. |
| F | Series resonance frequency (MHz). |
| FP | Parallel resonance frequency (MHz). |
| SEC | Time in seconds or any other data identification variable. |
| ØT | An input variable carried over from the input data to the new punched cards and printed listing. |

The following cards are called by the program:

| Card No. | Format | Variables |
|---|---|---|
| 1 | 3F10.2,I5,E15.7,2F10.2 | R, AL, PERCNT, LIMIT, CC, S, TSTART |
| 2 | 13A6, A2 | FMT |
| 3 | FMT | F, FP, SEC, ØT |
| 4 | FMT | F, FP, SEC, ØT |
| | etc. | |

The following routines are called:

| Subroutine Name | Location | Function |
|---|---|---|
| CRDIDS | System | Identify punched cards |
| TEMPSP | External | Calculate electron temperature |
| CRDIDF | System | End card punching |

Subroutine TEMPSP

Subroutine TEMPSP calculates the electron temperature from the series and parallel resonance frequencies of an antenna. Sheath effects can be taken into account by specifying the width of the sheath.

The calculation begins by assuming an initial electron temperature. This is provided externally along with the resonance frequencies and antenna parameters. The Debye length is then calculated from these values. The Debye length is then used to find the capacitance between the antenna and the outside edge of the sheath. The value of $\phi$ in the Despain theory can now be calculated. External subroutine ETTA calculates the argument of $\phi$ from the value of $\phi$. The electron temperature can then be determined from this argument ($\alpha' R$ in the Despain theory). This value of temperature is compared to the initial value, and if it is within the set error limits, the value of temperature is printed and punched and a new set of data is considered. If the temperatures differ more than the allowed error, the initial test temperature is set equal to the newly calculated temperature, and the calculation is repeated.

The variables for this routine are transferred through the labeled common block /KEN/ and they are defined above in connection with program TEP601.

Subroutine ETAA

ETAA calculates the inverse function $\eta$ of the $\phi$ ($\alpha' R$) of the Despain theory for real numbers only. Given the value of $\phi$, the subroutine determines $\alpha' R$ by a iteration process. A value of $\alpha' R$ is assumed. The corresponding value of $\phi$ is then determined. This value is compared to

the original value of $\phi$, and according to the difference between the values, the program either returns a value for $\alpha'R$ or makes a better estimate of $\alpha'R$ and repeats the process until the difference is less than .00001.

ETAA calls subroutine K(N,X) discussed above in connection with main program TMPTC1.

## Main Program IMPPLT

IMPPLT is the main program for plotting antenna resistance versus reactance with electron temperature and plasma frequency as parameters. The actual antenna impedance is calculated in subroutine ANTZ. As discussed above, several versions of ANTZ have been developed and any of these can be used in this main program. The program variables are:

| Variable Name | Function |
|---|---|
| NPLØTS | The number of separate figures to be produced. |
| NRESLV | The number of calculated points per curve. |
| NTAPE, NSKIP | Dummy variables, may be any integer. |
| NPRINT | The desired ratio of calculated and plotted points to the number of points printed on the tabular listing. |
| MPLT | The desired ratio of calculated and plotted points to the number of points indicated by a short line segment drawn normal to the curve and through the indicated point. |
| MTHY | A branching constant such that if it is set to 1, the electron collision frequency is specified by the altitude and electron temperature, otherwise collision frequency is taken from the input data. |

| Variable Name | Function |
|---|---|
| NDØ, KGRID | Dummy variables, may be any integer. |
| FMAX | The maximum value of plasma frequency (MHz) to be plotted. |
| FX | The value of the antenna exciting frequency (MHz). |
| FH | Electron gyrofrequency (Hz). |
| FNU | Electron collision frequency (Hz). |
| R | Antenna radius. |
| AL | Antenna length. |
| ANGLE | Magnetic aspect angle (degrees). |
| FNUCØN | A constant that relates collision frequency to altitude and electron temperature. |
| DEN | Total particle density of atmosphere at the given altitude. |
| ALT | Altitude |

The following cards are called by the program:

| Card No. | Format | Variables |
|---|---|---|
| 1 | 7I10 | NPLØTS, NRESLV, NTAPE, NSKIP, NPRINT, NPLT, MTHY |
| 2 | 2I10, 2F10.3 | DNØ, KGRID, FMAX, FX |
| 3 | 6E13.6 | FH, FNU, T, R, AL, ANGLE |
| 4 | 2E20.8, F10.3 | FNUCØN, DEN, ALT |
| 5,6,7,8,9 | are called by subroutine GRID discussed above. | |

Cards 2 through 9 are called once for each individual figure. The number of figures is specified by NPLØTS.

The following routines are called by IMPPLT:

| Subroutine Name | Location | Function |
|---|---|---|
| CØNSTS | External | Calculate physical constants |
| GRID | External | Generate a plot grid |
| ANTZ | External | Calculate antenna impedance |
| PLØT | System | Generate plot pen movements |
| FINI | System | End plotting |

## MAIN PROGRAM FOR PLOTTING ANTENNA IMPEDANCE AS A FUNCTION OF FREQUENCY

```
$IBFTC SWP4MD
C   PROGRAM FOR PLOTTING ANTENNA IMPEDANCE
      DIMENSION XX(1510), HEAD(50), XYZ(50), XRI(1510)
      COMPLEX Z,ZFIRST,ZSERIE,ZSHUNT,ZPAR,ZINTSH
      COMMON/IMPCOM/Z,F,FP,FNU,FH,ANGLE,T,R,AL
      LOGICAL NTFRST , UTAPE , FIRST
      UTAPE = .FALSE.
      CALL CONSTS
      READ 1, NPLOTS,NRESLV, NTAPE, NSKIP, NPRINT, MPLT, MTHY
      PRINT 1, NPLOTS, NRESLV, NTAPE, NSKIP, NPRINT, MPLT, MTHY
    1 FORMAT(7I10)
      NR1 = NRESLV +1
      NRX = 1500/NRESLV
      IF((NTAPE.GE.0).AND.(NTAPE.LE.4))UTAPE=.TRUE.
      IF(.NOT.UTAPE) GO TO 6
    6 REWIND NTAPE
    7 CALL SKPFLS(NTAPE,NSKIP,0)
    8 DO 113 JNP =1, NPLOTS
      NTFRST = .FALSE.
      READ 9, (HEAD(J),J=1,13)
      READ 9, (HEAD(J),J=14,26)
    9 FORMAT(13A6)
      READ 10,NDD,KGRID,FMAX,FX,RINIT,XINIT,CSHUNT,MODE
      PRINT 3,NDD,KGRID,FMAX,FX,RINIT,XINIT,CSHUNT,MODE
   10 FORMAT (2I10,2F10.3,3E12.5,I4)
    3 FORMAT(1H1,2I10,2F10.3,3E12.6,I4)
      FXM = FMAX* 1.0E+06
      FXX = FX* 1.0E+06
      SCALE = 15.0/FMAX
      SCXCON = .1E+05/SCALE
   11 FP = FXX
   12 F = FXX
   13 DO111 M=1, NDD
      READ 14,FH,FNU,T,R,AL,ANGLE
      PRINT 2, FH,FNU,T,R,AL,ANGLE
    2 FORMAT( 1H1,6E13.6)
   14 FORMAT(6E13.6)
      FHX=FH*1.0E-06
      FNUX=FNU*1.0E-06
      IF(UTAPE) WRITE(NTAPE,15) F,FP,FXM,FH,FNU
      IF(UTAPE) WRITE(NTAPE,15) T,R,AL,ANGLE,SCALE
   15 FORMAT(5E16.8)
      KPAGE = 52
      JK = NPRINT
      IF(MTHY.EQ.2) GO TO 26
   16 DO 25 K = 1, NR1
      AK = (K-1)*(NRX)
      IFPNT = AK/10.0
      F = AK*SCXCON
      ZSHUNT=CMPLX(0.0,(-1.0/CSHUNT*F*PI2))
      ZSERIE=CMPLX(RINIT,XINIT*F*PI2)
      FPNT = F*.1E-05
      CALL ANTZ
      GO TO (551,552,553,552),MODE
  551 RAA=REAL(Z)+RINIT
```

```
      XAA=AIMAG(Z)+XINIT*F
  552 GO TO 67
  553 ZPAR=Z*ZSHUNT/(Z+ZSHUNT)
      RAA=REAL(ZPAR)+REAL(ZSERIE)
      XAA=AIMAG(ZPAR)+AIMAG(ZSERIE)
  554 GO TO 67
   67 RA = RAA
      XA = XAA
      AM =1.0
      IF(XA.EQ.0.0) XA=1.0E-37
      IF(XA.GE.0.0)GO TO 17
      AM = -1.0
      XA = -XA
   17 XT = ALOG10(XA)
      IF (MODE.EQ.1) GO TO 68
      IF(XT.LT.(-2.0)) XT=0.0
      XX(K)=XT*AM
      GO TO 69
   68 IF(XT.LT.0.0)XT =0.0
      XX(K) = XT*AM/2.0
   69 IX = 1000.0*XX(K)
      AR= +1.0
      IF(RA.EQ.0.0)RA=1.0E-37
      IF(RA.GE.0.0)GO TO 18
      AR = -1.0
      RA = -RA
   18 RT = ALOG10(RA)
      IF (MODE.EQ.1) GO TO 70
      IF(RT.LT.(-2.0)) RT=0.0
      XR(K)=RT*AR
      GO TO 71
   70 IF(RT.LT.0.0) RT=0.0
      XR(K) = RT*AR/2.0
   71 IR = 1000.0*XR(K)
      IF(UTAPE) WRITE (NTAPE, 5)  IFPNT, IR ,IX,RAA,XAA, F
    5 FORMAT (3I10,2X,3E16.8)
      JK = JK+1
      JJ = JK/NPRINT
      IF(JJ.LT.1)GO TO 25
      KPAGE = KPAGE + 1
      IF(KPAGE.LE.50)GO TO 23
      KPAGE = 1
      PRINT 19
   19 FORMAT(1H1)
      PRINT 20
   20 FORMAT(1H ,132H--------------------------------------------------
     1-----------------------------------------------------------------
     2---------------      )
      PRINT    4, FX, FHX, FNUX, ANGLE, T,R,AL
    4 FORMAT( 2X,4HFP =,F5.2,14H  MHZ   ,  WH =,F4.2, 14H  MHZ   ,  NU =,
     1F7.4, 17H  MHZ   ,  ANGLE =,F5.2, 13H  DEG   ,  T =, F7.1, 14H DEG K
     2  ,  R =, F7.5, 11H M    ,  L =, F5.2, 2H M)
      PRINT 37,(HEAD(J),J=1,13)
   37 FORMAT(1H ,2X,13A6,50H -   IMPEDANCE AS A FUNCTION OF DRIVING FREQ
     1UENCY          )
```

```
      PRINT 20
      PRINT 20
      PRINT 22
   22 FORMAT(110H FREQUENCY (MHZ)     RESISTANCE (OHMS)          REACTANCE
     1 (OHMS)           PLOTX (INCHES)       PLOTR (INCHES)
      PRINT 20
   23 PRINT 24, FPNT,RAA,XAA,XR(K),XX(K)
   24 FORMAT(1H ,F10.3,2E25.8,2F20.3)
      JK=0
   25 CONTINUE
      GO TO 36
   26 ZSERIE=CMPLX(RINIT,XINIT*FXX*P(2)
      ZSHUNT=CMPLX(0.0,(-1.0/(CSHUNT*FXX*P(2)))
      DO 35 K = 1, NRI
      AK = (K-1)*(NRX)
      IFPNT = AK/10.0
      FP = AK*SCXCON
      FPNT = FP*.1E-05
      CALL ANTZ
      IF(K.EQ.1) ZFIRST=Z-ZSERIE
      GO TO (441,442,443,444),MODE
  441 RAA=REAL(Z)+RINIT
      XAA=AIMAG(Z)+XINIT*FXX
      GO TO 87
  442 RAA=REAL(Z)-REAL(ZFIRST)
      XAA=AIMAG(Z)-AIMAG(ZFIRST)
      GO TO 87
  443 ZPAR=Z*ZSHUNT/(Z+ZSHUNT)
      RAA =REAL(ZPAR)+REAL(ZSERIE)
      XAA=AIMAG(ZPAR)+AIMAG(ZSERIE)
      GO TO 87
  444 ZPAR=Z*ZSHUNT/(Z+ZSHUNT)
      IF(K.EQ.1) ZINTSH=ZPAR
      RAA =REAL(ZPAR)-(REAL(ZINTSH)-REAL(ZSERIE))
      XAA=AIMAG(ZPAR)-(AIMAG(ZINTSH)-AIMAG(ZSERIE))
   87 RA = RAA
      XA = XAA
      AM =1.0
      IF(XA.EQ.0.0) XA=1.0E-37
      IF(XA.GE.0.0)GO TO 27
      AM = -1.0
      XA = -XA
   27 XT = ALOG10(XA)
      IF(MODE.EQ.1) GO TO 78
      IF(XT.LT.(-2.0)) XT=0.0
      XX(K)=XT*AM
      GO TO 79
   78 IF(XT.LT.0.0)XT =0.0
      XX(K) = XT*AM/2.0
   79 IX = 1000.0*XX(K)
      AR= +1.0
      IF(RA.EQ.0.0)RA=1.0E-37
      IF(RA.GE.0.0)GO TO 28
      AR = -1.0
      RA = -RA
```

```
 28 RT = ALOG10(RA)
    IF(MODE.EQ.1) GO TO 80
    IF(RT.LT.(-2.0)) RT=0.0
    XR(K)=RT*AR
    GO TO 81
 80 IF(RT.LT.0.0) RT=0.0
    XR(K) = RT*AR/2.0
 81 IR = 1000.0*XR(K)
    IF(UTAPE) WRITE(NTAPE,5)  IFPNT,IR,IX,RAA,XAA,FP
    JK = JK+1
    JJ = JK/NPRINT
    IF(JJ.LT.1)GO TO 35
    KPAGE = KPAGE + 1
    IF(KPAGE .LE.50) GO TO 33
    KPAGE = 1
    PRINT 19
    PRINT 20
    PRINT 21,(HEAD(J),J=1,13)
 21 FORMAT(1H , 2X,13A6,48H - IMPEDANCE AS A FUNCTION OF PLASMA FREQUE
   1NCY  )
    PRINT  666, FX, FHX, FNUX, ANGLE, T,R,AL
666 FORMAT( 2X,4HFO =,F5.2,14H  MHZ   ,  WH =,F4.2, 14H  MHZ   ,  NU =,
   1F7.4, 17H  MHZ   ,  ANGLE =,F5.2, 13H  DEG   ,  T =, F7.1, 14H DEG K
   2  ,  R =, F7.5, 11H M    ,  L =, F5.2, 2H M)
    PRINT 20
    PRINT 20
    PRINT 22
    PRINT 20
 33 PRINT 24, FPNT,RAA,XAA,XR(K),XX(K)
    JK=0
 35 CONTINUE
 36 IF(UTAPE)END FILE NTAPE
    IF(MPLT.EQ.0)GO TO 111
    IF(NTFRST)GO TO 106
    SCFP = SCALE*FX
    W1B=(SCALE*FX-1.5)/2.0
    W1E=W1B+1.5
    W2B = (13.5+SCALE*FX)/2.0
    W3B = W2B + .05
    A1B=W1B+1.55
    A2B=W2B+1.6
    YYS = 0.0
    YYM = 0.0
    KI = K
    CALL IDPLOT
    CALL PLOT (2.0,5.3,-3)
    FPP=FX*SCALE
    L = 150./SCALE
    LL = L/10
    DO100J=1,L
    AJ = J
    AJ = AJ*SCALE/10.0
    CALL PLOT (AJ,0.0,2)
    CALL PLOT (AJ,+0.1,3)
    CALL PLOT (AJ,-0.1,2)
```

```
100 CALL PLOT (AJ,0.0, 3)
    IF(MPLT.NE.1) GO TO 101
    CALL PLOT (FPP,+0.1,3)
    CALL PLOT (FPP,-0.1,2)
    CALL SYMBL4(FPP-.05,+0.25,0.15,1HF,0.0,1)
    CALL SYMBL4(FPP+.02,+0.2,0.1,1HP,0.0,1)
101 DO 102J =1,LL
    AA = LL+1-J
    BJ = AA*SCALE
    CALL PLOT(BJ,+.15,3)
    CALL PLOT(BJ,-.15,2)
102 CALL NUMBER(BJ-0.17,-0.30,0.15,AA,0.0,1)
    IF(MPLT.NE.1) GO TO 103
    CALL SYMBL4(6.35,-.65,0.2,12HFREQUENCY-MC,0.0,12)
103 CALL PLOT (0.0,-4.0,3)
    IF(MODE .NE.1) GO TO 222
    DO 105J =1,17
    JC = J-9
    CJ = JC
    CY = CJ/2.0
    CZ = CJ
    IF(CJ.LT.0.0) CZ = -CJ
    CALL PLOT(0.0, CY ,2)
    CALL PLOT(+0.1,CY ,2)
    CX = +10.0
    IF(CJ.LT.0.0) GO TO 114
     CALL NUMBER(-0.1,CY-0.1 ,.15,CX  ,90.0,0)
104 CALL NUMBER(-.25,CY+0.1,0.1,CZ,90.0,0)
105 CALL PLOT (0.0,CY,3)
    GO TO  406
222 DO 405 JJJ=1,13
    JC=JJJ-7
    CJ=JC
    CY=CJ*.66666666
    CZ=CJ
    IF(CJ.LT.0.0) CZ=-CJ
    CZ=CZ-2.0
    CALL PLOT(0.0, CY ,2)
    CALL PLOT(+0.1,CY ,2)
    CX = +10.0
    IF(CJ.LT.0.0) GO TO 114
     CALL NUMBER(-0.1,CY-0.1 ,.15,CX  ,90.0,0)
223 CALL NUMBER(-.25,CY+0.1,0.1,CZ,90.0,0)
405 CALL PLOT (0.0,CY,3)
406 CALL SYMBL4(-.5,-1.8,.2,24HANTENNA IMPEDANCE (OHMS),90.0,24)
    CALL SYMBL4(0.0 ,-4.5 , 0.2,HEAD(1),0.0,78)
    CALL SYMBL4(3.35,-4.7 , 0.1,HEAD(14),0.0,78)
    CALL PLOT(+0.0,-4.0,3)
    CALL PLOT(15.0,-4.0,2)
    DO 118 J = 1 , 17
    JC = J - 9
    CJ = JC
    CY = CJ/2.0
    CALL PLOT(15.0,CY,2)
    CALL PLOT(14.9,CY,2)
```

```
118 CALL PLOT(15.0,CY,3)
    CALL PLOT( 0.0,+4.0,2)
106 NTFRST =.TRUE.
    FIRST  =.TRUE.
    DO 109 K = 1, NR1
    AK = (K-1)*NRX
    AK = AK/100.0
    XY = XX(K)
    IF(XY.GT.4.0) XY = 4.0
    IF(XY.LT.(-4.0)) XY = -4.0
    IF ( FIRST) CALL PLOT(AK,XY,3)
    FIRST = .FALSE.
    CALL PLOT(AK,XY,2)
    IF(MPLT.NE.1) GO TO 109
    IF(AK.LT.W1E) GO TO 107
    IF(AK.GT.A1B) GO TO 107
    IF(XY.GT.YYS)YYS = XY
    IF(XY.LT.YYM)YYM = XY
107 IF(AK.LT.W2B) GO TO 108
    IF(AK.GT.W3B) GO TO 108
    IF(XY.LT.XYS)XYS = XY
    IF(XY.GT.XYM)XYM = XY
108 IF(KI.EQ.0) GO TO 109
    IF((XY.GE.0.0).AND.(XYY.LE.0.0))GO TO 115
    XYY = XY
109 CONTINUE
    FIRST = .TRUE.
    DO 110 K=1,NR1
    AK = (K-1)*NRX
    AK = AK/100.0
    RR = XR(K)
    IF(RR.GT.4.0) RR = 4.0
    IF(RR.LT.(-4.0)) RR = -4.0
    IF( FIRST ) CALL PLOT (AK,RR,3)
    FIRST = .FALSE.
    CALL PLOT (AK,RR,2)
    IF(MPLT.NE.1)GO TO 110
    IF(AK.LT.W2B) GO TO 110
    IF(AK.GT.W3B) GO TO 110
    IF(RR.GT.RYS)RYS = RR
    IF(RR.LT.RYM)RYM = RR
110 CONTINUE
111 CONTINUE
    IF(MTHY.NE.1)GO TO 112
    IF(MPLT.EQ.0) GO TO 113
    IF(MPLT.NE.1)GO TO 112
    IF(SCFP.LT.1.0)GO TO 112
    IF(YYS.GT.3.0)YSW = YYM - .5
    IF(YYS.LE.3.0)YSW = YYS + .5
    IF(YYS.LT.0.0)YSW = YYM - .5
    IF(YYS.LT.(-3.0))YSW = YYS + .5
    CALL SYMBL4(W1B,YSW,0.1,17HANTENNA REACTANCE,0.0,17)
    IF((SCFP-15.0).GT.(-1.6)) GO TO 112
    IF(XYS.LT.(-3.0))XSW = XYM + .5
    IF(XYS.GE.(-3.0))XSW = XYS - .5
```

```
      IF(XYS.GE.( 0.0))XSW = XYM + .5
      IF(XYS.GE.( 3.0))XSW = XYS - .5
      IF(RYS.GT.( 3.0))RSW = RYM - .5
      IF(RYS.LE.( 3.0))RSW = RYS + .5
      IF(RYS.LE.( 0.0))RSW = RYM - .5
      IF(RYS.LE.(-3.0))RSW = RYS + .5
      CALL SYMBL4( W2B,XSW,0.1,17HANTENNA REACTANCE,0.0,17)
      CALL SYMBL4(W2B,RSW,0.1,18HANTENNA RESISTANCE,0.0,18)
  112 CONTINUE
      CALL PLOT (18.0,-5.3,-3)
  113 CONTINUE
      IF(UTAPE)REWINDNTAPE
      IF(MPLT.EQ.0) STOP
      CALL FINI
      STOP
  114 CX = -10.0
      CALL NUMBER(-0.1,CY-.22,.15,CX,90.0,0)
      IF(MODE.NE.1) GO TO 223
      GO TO 104
  115 CALL SYMBL4(AK-.03,.17,.07, 3HT =,90.0,3)
      TT = T
      IF(T.EQ.0.0) TT = 1.0E-36
      KAT = ALOG10(TT)
      TAK = KAT +7
      TX =.17 + TAK*.05636363
      CALL SYMBL4(AK-.10,TX-.05,.15,1H.,90.0,1)
      CALL SYMBL4 ( AK - .03 , TX , .07 , 1HK, 90.0,1)
      CALL NUMBER(AK-.03,.41,.07,T,90.0,0)
      KI = 0
      CALL PLOT (AK,XY,3)
      GO TO 109
      END
```

## SUBROUTINE TO CALCULATE ANTENNA IMPEDANCE ACCORDING TO MAGNETO-IONIC THEORY

```
0 $IBFTC ANTZ      MAGNETO - IONIC THEORY
  C FUNCTION SUBPROGRAM FOR CALCLATION OF ANTENNA IMPEDANCE
        SUBROUTINE ANTZ
        COMPLEX U,U2, YU,D,Z
        COMMON/CONSTA/PI,PI2,EMASS,AMASS,Q,CK,EO,G,UO,ALPHAK
        COMMON/IMPCOM/Z,F,FP,FNU,FH,ANGLE,T,R,AL
        CL=ALOG(AL/R)
        IF(F.LE.0.0) GO TO 1
        WF=PI2*F
        CO=PI2*EO*AL/CL
        IF (FP.LE.0.0) GO TO 2
        F2=F**2
        FP2=FP**2
        X=FP2/F2
        Y=FH/F
        Z=FNU/WF
        U=(1.0,0.0)-(0.0,1.0)*Z
        Y2=Y*Y
        U2=U*U
        YU=Y2/U2
        VR=((G*CK*T)/(2.0*EMASS*R*R)*FP2)
        D=(0.0,1.0)*WF*CO*(U*(YU-1.0)+(1.-VR)*(X-(X*YU*SIN(ANGLE)**2)/2.))
        IF(REAL(D).EQ.0.0.AND.AIMAG(D).EQ.0.0) GO TO 11
        Z=U*(YU-1.0)/D
        RETURN
      1 Z=(0.0,-.1E+37)
        RETURN
      2 Z=-(0.0,1.0)/(WF*CO)
        RETURN
     11 PRINT 12,YU,U,X
     12 FORMAT(14H ERROR IN ANTZ , 5E16.8)
        RETURN
        END
```

## SUBROUTINE ANTZ ACCORDING TO BALMAIN THEORY

```
$IBFTC ANTZ     LIST
C   FUNCTION SUBPROGRAM FOR CALCULATION OF ANTENNA IMPEDANCE
      SUBROUTINE ANTZ
      COMPLEX U,A2,EKO,EKP,AA2,AA,FM,FR ,AR,J,TT,CSQRT,CLOG,TNMAG,AA1,Z
      COMMON/CONSTA/PI,PI2,EMASS,AMASS,Q,CK,EO,G,UO,ALPHAK
      COMMON/IMPCOM/Z,F,FP,FNU,FH,ANGLE,T,R,AL
      WF = PI2*F
      F2 = F*F
      FP2 = FP*FP
      X = FP2/F2
      Y = FH/F
      Z = FNU/WF
      U = (1.0,0.0) -(0.0,1.0)*Z
      EKO = (1.0,0.0) - X/U
      EKP = (1.0,0.0)-X*U/(U*U-Y*Y)
      AA2 = EKP/EKO
      TT =(0.0,0.0)
      IF(T.LT.1.0)GO TO 2
      A2 = (F2*ALPHAK/T)*(X-U)
      AR = R*CSQRT(A2)
      TT = (EKO - (1.0,0.0))/((2.0,0.0)*AR)
    2 AA = CSQRT(AA2)
      RAD = PI*ANGLE/180.
      SN2 = SIN( RAD )**2
      CN2 = COS( RAD )**2
      FM = CMPLX(SN2,0.0) + AA2*CMPLX(CN2,0.0)
      FR =CSQRT(FM)
      AA1 = (AA+FR)/((2.0,0.0)*FM)
      TNMAG = TT
      IF((REAL(AA1).EQ.0.0).AND.(AIMAG(AA1) .EQ.0.0))GO TO 3
      TNMAG = -CLOG(AA1)  +TT
    3 Z=AA*((ALOG(AL/R)-1.0)+TNMAG)/(((0.0,1.0)*EKP*FR)*(PI2*WF*EO*AL))
      RETURN
      END
```

## SUBROUTINE ANTZ ACCORDING TO WHALE THEORY

```
      SUBROUTINE ANTZ
      COMPLEX Z,ARC,A2C
      COMMON/CONSTA/PI,PI2,EMASS,AMASS,Q,CK,EO,G,HO,ALPHAK
       COMMON/IMPCOM/ Z,F,FP,FNU,FH,ANGLE,T,R,AL
      DIMENSION XRO(100), BKRO(100)
      LOGICAL FIRST
      REAL LAMD
      IF(FIRST) GO TO  11
      FIRST=.TRUE.
      DATA BKRO/.50,.52,.535,.55,.565,.58,.60,.61,.62,.635,.645,.655,.66
     15,.67,.675,.685,.695,.70,.705,.710,.715,.72,.725,.728,.73,.733,.73
     25,.738,.740,.745,.748,.750,.755,.758,.759,.760,.762,.763,.764,.765
     3,.77,.772,.77,.765,.755,.75,.74,.735,.72,.715,.70,.695,.665,.65,.6
     43,.615,.600,.580,.565,.555,.545,.53,.52,.51,.505,.495,.485,.475,.4
     565,.455,.450,.440,.435,.430,.42,.415,.410,.405,.402,.395,.390,.385
     6,.380,.375,.370,.368,.365,.360,.355,.350,.340,.325,.315,.305,.295,
     7.290,.280,.275,.270,.265/
      DATA XRO/.10,.11,.12,.13,.14,.15,.16,.17,.18,.19,.20,.21,.22,.23,.
     124,.25,.26,.27,.28,.29,.30,.31,.32,.33,.34,.35,.36,.37,.38,.39,.40
     2,.41,.42,.43,.44,.45,.46,.47,.48,.50,.55,.60,.65,.70,.75,.80,.85,.
     390,.95,1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4
     4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,3.4,3.5,3.6,3.7,3.8,3.9,4.0,4
     5.1,4.2,4.3,4.4,4.5,4.6,4.7,4.8,4.9,5.0,5.5,6.0,6.5,7.0,7.5,8.0,8.5
     6,9.0,9.5,10.0 /
   11 CL = ALOG(AL/R )
      IF(F.LE.0.0) GO TO 1
      WF =PI2*F
      CO = PI2*EO*AL/CL
      XO = -1.0/(WF*CO)
      IF(FP.LE.0.0) GO TO 2
      F2 = F**2
      FP2= FP**2
      X  = FP2/F2
      IF(X.EQ.1.0) GO TO 1
      ZX = XO/(1.0 - X)
      IF(T.LE.0.0) GO TO 3
      V =  SQRT(G*CK*T/EMASS)
      LAMD =625.0*SQRT(T)/FP
      RO  =R + 4.0*LAMD
      C2  =(CO/AL)**2
      IF(X.GE.1.0) GO TO 3
      A2 =(1.0 -X)*FP2*ALPHAK/T
      AR = SQRT(A2)*RO
      ZX4 =4.0*(ZX**2)
      CALL INTERP (XRO,BKRO,AR,WBETA,100)
      PVI=PI2*EO*RO*SQRT(1.0-X)/(X*V*C2*AL*WBETA**2)
      PVI2= PVI**2
      IF(ZX4.GT.PVI2)GO TO 3
      R1= .5*(PVI + SQRT(PVI2-ZX4))
      R2= .5*(PVI - SQRT(PVI2-ZX4))
      ZR =R1
      IF(R1.GT.R2)ZR=R2
      Z = CMPLX(ZR,ZX)
      RETURN
    1 Z = CMPLX(0.0,-1E+37)
      RETURN
    2 Z = CMPLX(0.0,XO)
      RETURN
    3 Z = CMPLX(0.0,ZX)
      RETURN
      END
```

## SUBROUTINE TO CALCULATE ANTENNA IMPEDANCE ACCORDING TO DESPAIN THEORY

```
$IBFTC ANTZ     DESPAIN THEORY  12/08/65
C  SUBROUTINE TO CALCULATE ANTENNA IMPEDANCE ACCORDING TO DESPAIN THEORY
      SUBROUTINE ANTZ
      COMPLEX U,A2,U2,S2,A2U2DW,AR,CSQRT,PHI,Z,XO
      COMMON/CONSTA/PI,PI2,EMASS,AMASS,Q,CK,EO,G,UO,ALPHAK
      COMMON/IMPCOM/Z,F,FP,FNU,FH,ANGLE,T,R,AL
      COMMON /FPHIC/AR,PHI
      CL=ALOG(AL/R)
      IF(F.LE.0.0) GO TO 1
      WF=PI2*F
      CO=PI2*EO*AL/CL
      XO=-(0.0,1.0)/(WF*CO)
      IF (FP.LE.0.0) GO TO 2
      F2=F**2
      FP2=FP**2
      X=FP2/F2
      Y=FH/F
      Z=FNU/WF
      U=(1.0,0.0)-(0.0,1.0)*Z
      Y2=Y*Y
      U2=U*U
      S2=(U2-Y2)/(U*X)
      IF(T.LE.0.0) GO TO 3
      A2=(S2-(1.0,0.0))*FP2*ALPHAK/T
      AR=R*CSQRT(A2)
    5 IF(REAL(AR)) 6,7,8
    6 IF(AIMAG(AR))11,17,10
    7 IF(AIMAG(AR))9,17,10
    8 IF(AIMAG(AR))9,9,11
    9 AR=-AR
   17 IF(AIMAG(AR).LE.0.0) AR=AR+(0.0,.1E-16)
   10 CALL APHICF
      Z=XO*(PHI/CL-S2)/((1.0,0.0)-S2)
      RETURN
    1 Z=(0.0,-.1E+37)
      RETURN
      Z=XO
      RETURN
    3 Z=-XO*S2/((1.0,0.0)-S2)
      RETURN
   11 PRINT 12,AR,FH,FINS,ALPHAK,T,R
   12 FORMAT(14H ERROR IN ANTZ , 7E16.8)
      RETURN
      END
```

## SUBROUTINE FOR CALCULATION OF PHYSICAL CONSTANTS

```
$IBFTC CONSTS
      SUBROUTINE CONSTS
      COMMON /CONSTA/PI,PI2,EMASS,AMASS,Q,CK,EO,G, UO,ALPHAK
      PI = 3.14159265
      PI2 =2.0*PI
      EMASS = 9.1083E-31
      AMASS = EMASS*1836.12*26.5
      Q  =1.60206E-19
      CK = 1.38044E-23
      EO = 8.85434E-12
      G  = 3.0
      UO = PI *4.0E-07
      ALPHAK = 4.0*PI*PI*EMASS/(G*CK)
      RETURN
      END
```

## INTERPOLATION SUBROUTINE

```
      SUBROUTINE INTERP(XA,YA,XIN,YOUT,NPTS)
      DIMENSION XA(100),YA(100)
      I=1
    1 IF(XA(I+1).GE.XIN.AND.XA(I).LT.XIN) GO TO 2
      IF(I.GE.(NPTS-2)) GO TO 2
      I=I+1
      GO TO 1
    2 YOUT=((XIN-XA(I+1))*(XIN-XA(I+2))*YA(I)/((XA(I)-XA(I+1))*(XA(I)-XA
     1(I+2))))+((XIN-XA(I))*(XIN-XA(I+2))*YA(I+1)/((XA(I+1)-XA(I))*(XA(I
     2+1)-XA(I+2))))+((XIN-XA(I))*(XIN-XA(I+1))*YA(I+2)/((XA(I+2)-XA(I+1
     3))*(XA(I+2)-XA(I))))
      RETURN
      END
```

## SUBROUTINE FOR CALCULATION OF PHI FUNCTION

```
$IBFTC APHICF  DECK
      SUBROUTINE APHICF
      LOGICAL NTFRST,VAL1,VAL2,VAL3,VAL4
      DOUBLE PRECISION XR,XI,PHIR,PHII,PI,PII,PII2,CPI2R,CPI2I,PI4I,PI3
     14,CP4R,CP4I,CP3R,CP3I,JR,JI,CMULTR,CMULTI,CDVDR,DCVDI,AK,AI,BR,BI
     2,YR,YI,Y2R,Y2I,BOR,BDI,B1R,B1I,BJOR,BJ0I,BJ1R,BJ1I,BNOR,BNDI,BN1R,
     3BN1I,TJOR,TJOI,TJ1R,TJ1I,TNOR,TNOI,TN1R,TN1I,F,S,BDRS,BOIS,B1RS,B1
     4IS,F1,SM,BN1RS,BN1IS,BJ1RS,BJ1IS,BFR,BFI,DLOG,DATAN2,HOR,HOI,H1R,H
     51I,XBR,XBI,XBQR,XBQI,POR,POI,QOR,QOI,P1R,P1I,Q1R,Q1I,TPOR,TPDI,TQD
     6R,TQDI,TP1R,TP1I,TQ1R,TQ1I,D,D2,DM2,D4,D4Q,D6,DQ,D2Q,    F2,XB1R,XB
     71I,XBQIR,XBQII,    RPO,RQO,RP1,RQ1,PORS,POIS,QORS,QOIS,P1RS,P1IS,Q
     81RS,Q1IS,QDRS1,QOIS1,Q1RS1,Q1IS1
      COMPLEX AR,PHI
      COMMON /FPHIC/AR,PHI
      XR=REAL(AR)
      XI=AIMAG(AR)
      VAL1=.FALSE.
      VAL2=.FALSE.
      VAL3=.FALSE.
      VAL4=.FALSE.
      IF(NTFRST) GO TO 1
      NTFRST=.TRUE.
      PI=3.1415926535897932
      PII=1.0D0/PI
      PII2=16.0D0*PII
      CPI2R=PII2
      CPI2I=0.0D0
      PI4I =PI/4.0D0
      PI34=PI4I*3.0D0
      CP4R=PI4I
      CP4I=0.0D0
      CP3R=PI34
      CP3I=0.0D0
      JR=0.0D0
      JI=1.0D0
      CMULTR(AK,AI,BR,BI)=(AK*BR-AI*BI)
      CMULTI(AK,AI,BR,BI)=(BR*AI+AK*BI)
      CDVDR(AK,AI,BR,BI)=(AK*BR+AI*BI)/(BR*BR+BI*BI)
      CDVDI(AK,AI,BR,BI)=(AI*BR-AK*BI)/(BR*BR+BI*BI)
    1 YR=CMULTR(.5D0,0.0D0,XR,XI)
      YI=CMULTI(.5D0,0.0D0,XR,XI)
      K=1
      IF((YR.EQ.0.0D0).AND.(YI.EQ.0.0)) K=2
      GO TO (3,4),K
    4 YR=.1D-17
      YI=0.0
    3 Y2R=CMULTR(YR,YI,YR,YI)
      Y2I=CMULTI(YR,YI,YR,YI)
      IF((XR**2+XI**2).GE.64.0D0) GO TO 13
      BOR=1.0D0
      BOI=0.0D0
      B1R=BOR
      B1I=BOI
      BJOR=BOR
      BJOI=BOI
```

```
      BJ1R=B0R
      BJ1I=B0I
      BN0R=0.0D0
      BN0I=0.0D0
      BN1R=B0R
      BN1I=B0I
      TJ0R=B0R
      TJ0I=B0I
      TJ1R=B0R
      TJ1I=B0I
      TN0R=BN0R
      TN0I=BN0I
      TN1R=B0R
      TN1I=B0I
      F=1.0D0
      S=0.0D0
      DO  9 K=1,100
      B0RS=B0R
      B0IS=B0I
      B1RS=B1R
      B1IS=B1I
      F1=F+1.0D0
      B0R=-(B0RS*Y2R-B0IS*Y2I)/(F*F)
      B0I=-(B0IS*Y2R+B0RS*Y2I)/(F*F)
      B1R=-(B1RS*Y2R-B1IS*Y2I)/(F*F1)
      B1I=-(B1IS*Y2R+B1RS*Y2I)/(F*F1)
      S=S+1.0D0/F
      SM=2.0D0*S +1.0D0/F1
      BJ0R=BJ0R+B0R
      BJ0I=BJ0I+B0I
      BJ1R=BJ1R+B1R
      BJ1I=BJ1I+B1I
      BN0R=BN0R+B0R*S
      BN0I=BN0I+B0I*S
      BN1R=BN1R+B1R*SM
      BN1I=BN1I+B1I*SM
      IF(BJ0R.NE.TJ0R.OR.BJ0I.NE.TJ0I) GO TO 8
      IF(BJ1R.NE.TJ1R.OR.BJ1I.NE.TJ1I) GO TO 7
      IF(BN0R.NE.TN0R.OR.BN0I.NE.TN0I) GO TO 6
      IF(BN1R.NE.TN1R.OR.BN1I.NE.TN1I) GO TO 5
      GO TO 10
    5 TN1R=BN1R
      TN1I=BN1I
    6 TN0R=BN0R
      TN0I=BN0I
    7 TJ1R=BJ1R
      TJ1I=BJ1I
    8 TJ0R=BJ0R
      TJ0I=BJ0I
    9 F=F1
   10 BN0R=2.0D0*BN0R
      BN0I=2.0D0*BN0I
      BN1RS=BN1R
      BN1IS=BN1I
      BJ1RS=BJ1R
```

```
      BJ1IS=BJ1I
      BN1R=CMULTR(BN1RS,BN1IS,YR,YI)+CDVDR( 1.0D0,0.0D0,YR,YI)
      BN1I=CMULTI(BN1RS,BN1IS,YR,YI)+CDVDI( 1.0D0,0.0D0,YR,YI)
      BJ1R=CMULTR(BJ1RS,BJ1IS,YR,YI)
      BJ1I=CMULTI(BJ1RS,BJ1IS,YR,YI)
      BFR=2.0D0*(.5772156649015328 +.5D0*DLOG(YR*YR+YI*YI))
      BFI=2.0D0*DATAN2(YI,YR)
      BNOR=(CMULTR(BFR,BFI,BJOR,BJOI)-BNOR)*PII
      BNOI=(CMULTI(BFR,BFI,BJOR,BJOI)-BNOI)*PII
      BN1R=(CMULTR(BFR,BFI,BJ1R,BJ1I)-BN1R)*PII
      BN1I=(CMULTI(BFR,BFI,BJ1R,BJ1I)-BN1I)*PII
   11 HOR=BJOR+CMULTR(JR,JI,BNOR,BNOI)
      HOI=BJOI+CMULTI(JR,JI,BNOR,BNOI)
      H1R=BJ1R+CMULTR(JR,JI,BN1R,BN1I)
      H1I=BJ1I+CMULTI(JR,JI,BN1R,BN1I)
      PHIR=CDVDR(HOR,HOI,(CMULTR(CMULTR(YR,YI,H1R,H1I),CMULTI(YR,YI,H1R,
     1H1I),2.0D0,0.0D0)),(CMULTI(CMULTR(YR,YI,H1R,H1I),CMULTI(YR,YI,H1R,
     2H1I),2.0D0,0.0D0)))
      PHII=CDVDI(HOR,HOI,(CMULTR(CMULTR(YR,YI,H1R,H1I),CMULTI(YR,YI,H1R,
     1H1I),2.0D0,0.0D0)),(CMULTI(CMULTR(YR,YI,H1R,H1I),CMULTI(YR,YI,H1R,
     2H1I),2.0D0,0.0D0)))
      PHI= CMPLX(PHIR,PHII)
      RETURN
   13 XBR=CMULTR(16.0D0,0.0D0,YR,YI)
      XBI=CMULTI(16.0D0,0.0D0,YR,YI)
      XBQR=CMULTR(XBR,XBI,XBR,XBI)
      XBQI=CMULTI(XBR,XBI,XBR,XBI)
      M=2
      IF((XBQR.EQ.0.0).AND.(XBQI.EQ.0.0))M=1
      GO TO (14,15),M
   14 XBQR=.1D-17
      XBQI=0.0D0
   15 POR=1.0D0
      POI=0.0D0
      QOR=POR
      QOI=POI
      P1R=POR
      P1I=POI
      Q1R=3.0D0
      Q1I=0.0D0
      TPOR=POR
      TPOI=POI
      TQOR=POR
      TQOI=POI
      TP1R= POR
      TP1I=POI
      TQ1R=Q1R
      TQ1I=Q1I
      D=1.0D0
      F=1.0D0
      XBIR=CDVDR(1.0D0,0.0D0,XBR,XBI)
      XBII=CDVDI(1.0D0,0.0D0,XBR,XBI)
      XBQIR=CDVDR(1.0D0,0.0D0,XBQR,XBQI)
      XBQII=CDVDI(1.0D0,0.0D0,XBQR,XBQI)
      DO 20 K=1,500
```

```
      DM2=D-2.0D0
      D2=D+2.0D0
      D4=D+4.0D0
      D4Q=D4*D4
      D6=D+6.0D0
      DQ=D*D
      D2Q=D2*D2
      F1=F+1.0D0
      F2=F+2.0D0
      RP0=-D2Q*DQ/(F*F1)
      RQ0=-D2Q*D4Q/(F1*F2)
      RP1=-DM2*D*D2*D4/(F*F1)
      RQ1=-D*D2*D4*D6/(F1*F2)
      P0RS=((X8Q1R*RP0)*TP0R-(X8Q1I*RP0)*TP0I)
      P0IS=((X8Q1R*RP0)*TP0I+(X8Q1I*RP0)*TP0R)
      Q0RS=((X8Q1R*RQ0)*TQ0R-(X8Q1I*RQ0)*TQ0I)
      Q0IS=((X8Q1R*RQ0)*TQ0I+(X8Q1I*RQ0)*TQ0R)
      P1RS=((X8Q1R*RP1)*TP1R-(X8Q1I*RP1)*TP1I)
      P1IS=((X8Q1R*RP1)*TP1I+(X8Q1I*RP1)*TP1R)
      Q1RS=((X8Q1R*RQ1)*TQ1R-(X8Q1I*RQ1)*TQ1I)
      Q1IS=((X8Q1R*RQ1)*TQ1I+(X8Q1I*RQ1)*TQ1R)
      IF(VAL1) GO TO 32
      IF((Q1RS*Q1RS+Q1IS*Q1IS).LE.(TQ1R*TQ1R+TQ1I*TQ1I)) GO TO 19
      VAL1=.TRUE.
   32 IF(VAL2) GO TO 33
      IF((P1RS*P1RS+P1IS*P1IS).LE.(TP1R*TP1R+TP1I*TP1I)) GO TO 18
      VAL2=.TRUE.
   33 IF(VAL3) GO TO 34
      IF((Q0RS*Q0RS+Q0IS*Q0IS).LE.(TQ0R*TQ0R+TQ0I*TQ0I)) GO TO 17
      VAL3=.TRUE.
   34 IF(VAL4) GO TO 35
      IF((P0RS*P0RS+P0IS*P0IS).LE.(TP0R*TP0R+TP0I*TP0I)) GO TO 16
      VAL4=.TRUE.
   35 IF(VAL1.AND.VAL2.AND.VAL3.AND.VAL4) GO TO 21
      GO TO 693
   16 P0R=P0RS+P0R
      P0I=P0IS+P0I
      IF(TP0R.EQ.P0R.AND.TP0I.EQ.P0I) VAL4=.TRUE.
      TP0R=P0RS
      TP0I=P0IS
      GO TO 35
   17 Q0R=Q0RS+Q0R
      Q0I=Q0IS+Q0I
      IF(TQ0R.EQ.Q0R.AND.TQ0I.EQ.Q0I) VAL3=.TRUE.
      TQ0R=Q0RS
      TQ0I=Q0IS
      GO TO 34
   18 P1R=P1RS+P1R
      P1I=P1IS+P1I
      IF(TP1R.EQ.P1R.AND.TP1I.EQ.P1I) VAL2=.TRUE.
      TP1R=P1RS
      TP1I=P1IS
      GO TO 33
   19 Q1R=Q1RS+Q1R
      Q1I=Q1IS+Q1I
```

```
      IF(TQ1R.EQ.Q1R.AND.TQ1I.EQ.Q1I) VAL1=.TRUE.
      TQ1R=Q1RS
      TQ1I=Q1IS
      GO TO 32
  693 D=D4
   20 F=F2
   21 Q0RS1=Q0R
      Q0IS1=Q0I
      Q1RS1=Q1R
      Q1IS1=Q1I
      Q0R=-CMULTR(Q0RS1,Q0IS1,X8IR,X8II)
      Q0I=-CMULTI(Q0RS1,Q0IS1,X8IR,X8II)
      Q1R=CMULTR(Q1RS1,Q1IS1,X8IR,X8II)
      Q1I=CMULTI(Q1RS1,Q1IS1,X8IR,X8II)
      PHIR =CDVDR(CMULTR(JR,JI,(P0R+CMULTR(JR,JI,Q0R,Q0I)),(P0I+CMULTI(J
     1R,JI,Q0R,Q0I))),(CMULTI(JR,JI,(P0R+CMULTR(JR,JI,Q0R,Q0I)),(P0I+CMU
     2LTI(JR,JI,Q0R,Q0I)))),CMULTR(2.0D0,0.0D0, CMULTR(YR,YI,(P1R+CMULTR
     3(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,JI,Q1R,Q1I)))),CMULTI(YR,YI,(P1R+CM
     4ULTR(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,JI,Q1R,Q1I))))),CMULTI(2.0D0,0
     5.0D0,CMULTR(YR,YI,(P1R+CMULTR(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,JI,Q1
     6R,Q1I))),CMULTI(YR,YI,(P1R+CMULTR(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,J
     7I,Q1R,Q1I)))))
      PHII =CDVDI(CMULTR(JR,JI,(P0R+CMULTR(JR,JI,Q0R,Q0I)),(P0I+CMULTI(J
     1R,JI,Q0R,Q0I))),(CMULTI(JR,JI,(P0R+CMULTR(JR,JI,Q0R,Q0I)),(P0I+CMU
     2LTI(JR,JI,Q0R,Q0I)))),CMULTR(2.0D0,0.0D0, CMULTR(YR,YI,(P1R+CMULTR
     3(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,JI,Q1R,Q1I)))),CMULTI(YR,YI,(P1R+CM
     4ULTR(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,JI,Q1R,Q1I))))),CMULTI(2.0D0,0
     5.0D0,CMULTR(YR,YI,(P1R+CMULTR(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,JI,Q1
     6R,Q1I))),CMULTI(YR,YI,(P1R+CMULTR(JR,JI,Q1R,Q1I)),(P1I+CMULTI(JR,J
     7I,Q1R,Q1I)))))
      PHI= CMPLX(PHIR,PHII)
      RETURN
      END
```

## MAIN PROGRAM FOR CALCULATING AND PLOTTING SERIES AND PARALLEL RESONANCE FREQUENCIES WITH ELECTRON TEMPERATURE AS A PARAMETER.

```
$IBFTC TMPTC1
      COMMON /INPTS/ T(10),NT,FP(1500),NFP,R,AL,SI(1500),SS(10,1500),
     1 SHEATH,FH,FS(1500)
      DO 6 KKK=1,6
      CALL INPUT
      CALL GRID(6HVREADR,SCALEX,SCALEY,XB,XE,YB,YE,NX,NY,X'5)
      PRINT 3,(T(K),K=1,10)
      PRINT 4,NT,NFP,NX,NY,(FP(K),K=1,5)
      PRINT 3,SCALEX,SCALEY,XB,XE,YB,YE,X15,R,AL,SHEATH,FH
      DO 2  J =1,NT
      CALL SICALC(T(J),R,AL,FP,SI,NFP,SHEATH,FH)
      DO 1  K =1,NFP
      FS(K)=SQRT(FH**2+SI(K)**2*(FP(K)**2-FH**2))
    1 SS(J,K) =S(K)
      CALL CURVEL(FP,FS,NFP,XB,XE,YB,YE,X15)
      FFF=(FS(NFP)-YB)*9.0/(YE-YB)
      FF=(FP(NFP)-XB)*X15/(XE-XB)
      AJ=J-1
      CALL ARROW (FF,FFF,0.0 )
      CALL PLOT ((FF+1.0),FFF,2)
      CALL PLOT((FF+2.0),AJ,2 )
      CALL PLOT ((FF+3.0),AJ,2)
    2 CALL NUMBER((FF+1.75),AJ,.125,T(J),0.0,0)
    3 FORMAT(1H ,11E12.6)
    4 FORMAT(1H ,4(10,5E16.8)
      CALL OUTPUT
      CALL FINI
    6 CONTINUE
      CALL EXIT
      END
```

## SUBROUTINE FOR READING INPUT DATA FOR MAIN PROGRAM TMPTCI

```
$IBFTC INPUT
      SUBROUTINE INPUT
      COMMON /INPTS/ T(10),NT,FP(1500),NFP,R,AL,SI(1500),SS(10,1500),
     1 SHEATH,FH,FS(1500)
      READ 1,NT,NFP,R,AL,FPMAX,FPMIN,SHEATH,FH
    1 FORMAT(2I5,6E11.5)
      READ 2,(T(J),J =1,10)
    2 FORMAT( 5E16.8)
      AN = NFP -1
      DELF =(FPMAX-FPMIN)/AN
      DO 3 J = 1,NFP
      AJ=J-1
    3 FP(J)=FPMIN+AJ*DELF
      RETURN
      END
```

## SUBROUTINE FOR PRINTING DATA OUT OF MAIN PROGRAM TMPTC1

```
      SUBROUTINE OUTPUT
      COMMON /INPTS/ T(10),NT,FP(1500),NFP,R,AL,SI(1500),SS(10,1500),
     1 SHEATH,FH,FS(1500)
      DIMENSION I(10)
      LOGICAL FIRST
      FIRST =.TRUE.
      R = R*1.0E+02
   13 KPAGE = 52
      DO 3   J = 1,NFP
      KPAGE = KPAGE + 1
      IF(KPAGE.GT.50) GO TO 4
    1 PRINT 2 , FP(J),(SS(K,J),K =1,NT)
    2 FORMAT(1H ,F11.3,10F12.3)
      IF(.NOT.FIRST) GO TO 3
      DO 3 K=1,NT
      SS(K,J)=SQRT(FH**2+SS(K,J)**2*(FP(J)**2-FH**2))
    3 CONTINUE
      IF(.NOT.FIRST)RETURN
      FIRST =.FALSE.
      GO TO 13
    4 KPAGE=1
      PRINT 5
    5 FORMAT(1H1)
      PRINT 6
    6 FORMAT(1H ,132H---------------------------------------------------
     1-------------------------------------------------------------------
     2-------------- )
      IF(FIRST) PRINT  7
      IF(.NOT.FIRST)PRINT12
    7 FORMAT(1H , 90H
     1CYLINDRICAL ANTENNA PSI CALCULATIONS                                )
   12 FORMAT(1H,105H                                       CYLINDRICAL AN
     1TENNA CALCULATIONS OF SERIES RESONANCE  FREQUENCY                   )
      PRINT 8 ,R, AL,FH
    8 FORMAT(1H ,24X,10H RADIUS = ,F6.4,17H CM   ,  LENGTH = ,F6.4,
     128H METERS   ,  GYRO FREQUENCY =,F 5.3 ,11H MEGACYCLES )
      PRINT 6
      PRINT 9
    9 FORMAT( 92H PARALLEL FREQUENCY
     1            ELECTRON  TEMPERATURE                 )
      DO 10 L=1,10
   10 I(L) = T(L)
      PRINT 11,(I(L),L=1,10)
   11 FORMAT(16H   (MEGACYCLES) , I6,4H*K   , 9(I8,4H*K   ))
      PRINT 6
      GO TO 1
      END
```

## SUBROUTINE TO CALCULATE SERIES RESONANCE FREQUENCY

```
$IBFTC SICALC
      SUBROUTINE SICALC(T,R,AL,FP,SI,NFP,SHEATH,FH     )
      DIMENSION FP(1500) , SI(1500)
      COMMON /CONSTA/ PI,PI2,EMASS,AMASS,Q,CK,EO,G,UO,ALPHAK
      REAL K
      CL = ALOG(AL/R)
      CALL CONSTS
      DBYCON =SQRT(CK/(4.0*PI2*EMASS))*1.0E-06
      SI2 =.1
      DO 7  J =1,NFP
      L = NFP+1-J
      IF(FP(L).LE.FH) GO TO 10
      ALPFT =ALPHAK*(FP(L)**2-FH**2)/T
      DBYLNT = DBYCON*SQRT(T)/FP(L)
      RS = R + DBYLNT*SHEATH
      SHTCON = ALOG(RS/R)
      N = 0
      GO TO 3
    1 SI2=AN*.04
    2 SI2=.5*SI2
    3 S2 =1.0 -SI2
      N = N+1
      AN = N
      IF(N.GT.100) GO TO 8
      A2 = S2*ALPFT
      IF(A2.LE.1.0E-18) A2 = 1.0E-18
      AR = RS*SQRT(A2)
      RKAR = K(0,AR)/K(1,AR)
      RKM = 1.0-RKAR**2
      S22 = 2.0*S2
      SI2P = (SI2*RKM+S22*(SHTCON+RKAR/AR))/(RKM+S22*CL)
      SLOPE = 1.0 + RKM/(S22*CL)
      DEL  =(SI2P -SI2)
      IF(SLOPE)2,2,4
    4 IF(SI2P )2,2,5
    5 IF(SI2P- 1.0)6,1,1
    6 SI2 = SI2P
      IF(ABS(DEL).GT..001)GO TO 3
    7 SI(L) =SQRT(SI2)
      RETURN
    8 PRINT 9,DEL,SI2,AR,T,FP(J),SLOPE
    9 FORMAT(31H SICALC HAS FAILED TO COVERGE.  ,6E16.8)
      GO TO 7
   10 SI2=0.0
      GO TO 7
      END
```

**SUBFUNCTION PROGRAM FOR CALCULATION OF THE BESSEL FUNCTIONS $K_0$ AND $K_1$**

```
      REAL FUNCTION K(N,X)
      REAL I
      K = 0.0
      IF(N.LT.0) N =-N
      IF(N.GE.2) RETURN
      IF(X.EQ.0.0)GO TO 1
      IF(N.EQ.1) GO TO 3
      IF(X.GT.2.0) GO TO 2
      Y = X/2.0
      Y2 = Y*Y
      Y4 = Y2*Y2
      Y6 = Y4*Y2
      Y8 = Y4*Y4
      Y10 = Y6*Y4
      Y12 = Y6*Y6
      K   = -ALOG(Y)*I(N,X)-.57721566 +.42278420*Y2 +.23069756*Y4
     1+ .03488590*Y6 + .00262698*Y8 +.00010750*Y10 + .00000740*Y12
      RETURN
    1 K=1.0E+37
      RETURN
    2 Z = 2.0/X
      Z2 = Z*Z
      Z3 = Z2*Z
      Z4 = Z3*Z
      Z5 = Z4*Z
      Z6 = Z5*Z
      K = 1.25331414 -.07832358*Z + .02189568*Z2 - .01062446*Z3
     1+ .00587872*Z4 -.00251540*Z5 +.00053208*Z6
      K = K*EXP(-X)*(X**(-.5))
      RETURN
    3 IF(X.GT.2.0)GO TO 4
      Y = X/2.0
      Y2= Y*Y
      Y4= Y2*Y2
      Y6= Y4*Y2
      Y8= Y6*Y2
      Y10=Y8*Y2
      Y12=Y10*Y2
      K = ALOG(Y)*I(N,X) +(1.0 +.15443144*Y2 -.67278579*Y4-.18156897*Y6
     1- .01919402*Y8 - .00110404*Y10 - .0000468*Y12)/X
      RETURN
    4 Z = 2.0/X
      Z2= Z*Z
      Z3= Z2*Z
      Z4= Z3*Z
      Z5= Z4*Z
      Z6= Z5*Z
      K = 1.25331414 +.23498619*Z - .03655620*Z2 + .01504268*Z3
     1 - .00780353*Z4 + .00325614*Z5 - .00068245*Z6
      K = K*EXP(-X)*(X**(-.5))
      RETURN
      END
```

## SUBFUNCTION PROGRAM FOR THE CALCULATION OF THE BESSEL FUNCTIONS $I_0$ AND $I_1$

```
  REAL FUNCTION I(N,X)
  DOUBLE PRECISION Y,FACN,FACM,FJ,SUM,TERM,FK,FKN
  Y=X/2.0
  I =C.0
  IF(N.LT.0) N = -N
  FACN =1.0
  FACM =1.0
  IF(N.EQ.0) GO TO 4
  DC I J = 1,N
  FJ =J
1 FACN = FACN*FJ
  SUM =(Y**N)/FACN
  TERM = SUM
2 DO 3 K =1 , 500
  FK   = K
  FKN  = K + N
  TERM = TERM*Y*Y/(FKN*FK)
  SUM = SUM + TERM
  SS=SUM
  IF(I.EQ.SS ) RETURN
3 I = SS
  RETURN
4 SUM  = 1.0
  TERM = 1.0
  GO TO 2
  END
```

## SUBROUTINE FOR DRAWING A CURVE FROM AN ARRAY OF DATA

```
$IBFTC CURVEL
        SUBROUTINE CURVEL(AX,AY,N,XB,XE,YB,YE,X15)
        DIMENSION AX(10),AY(10)
        LOGICAL FIRST
        FIRST = .TRUE.
        DX =XE-XB
        DY =YE-YB
        SCX=X15/DX
        SCY=9.0/DY
        DO  1  J =1,N
        X =(AX(J)- XB)*SCX
        Y =(AY(J)- YB)*SCY
        IF(X.GT.X15) GO TO 3
        IF(X.LT.0.0) GO TO 3
        IF(Y.GT.9.0) GO TO 3
        IF(Y.LT.0.0) GO TO 3
        IF(FIRST)CALL PLOT(X,Y,3)
        CALL PLOT(X,Y,2)
        FIRST = .FALSE.
        GO TO 1
      3 FIRST = .TRUE.
      1 CONTINUE
        RETURN
        END
```

## SUBROUTINE TO PRODUCE A LABELED GRID FOR PLOTTED DATA

```
      SUBROUTINE GRID(TITLE,SCX,SCY,XXB,XXE,YYB,YYE,NXX,NYY,XX15)
C  ALTERNATE ARGUMENTS GRID(6H READ )  ,  GRID(6HVREADR) , GRID(6HCOMMON)
      DIMENSION  TITLE(52)
      COMMON/PLTCMN/HEAD(65),SCALEX,SCALEY,XB,XE,YB,YE,NX,NY,X15
      CALL HOL(CMN,6HCOMMON)
      IF(TITLE(1) .EQ.CMN) GO TO 106
      CALL HOL(READIN,6H READ )
      CALL HOL(READER,6HVREADR)
      IF((TITLE(1).NE.READER).AND.(TITLE(1).NE.READIN)) GO TO 104
      READ 101,(HEAD(J),J =1,52)
  101 FORMAT(13A6)
      READ 102, SCALEX,SCALEY,XB,XE,YB,YE,NX,NY,X15
  102 FORMAT( 6F10.5,2I5,1F10.5)
      PRINT 1000
 1000 FORMAT(1H1)
      PRINT 102,SCALEX , SCALEY,XB,XE,YB,YE,NX,NY,X15
      PRINT 1000
      IF(TITLE(1).EQ.READIN) GO TO 103
      SCX = SCALEX
      SCY = SCALEY
      XXB = XB
      XXE = XE
      YYB = YB
      YYE = YE
      NXX = NX
      NYY= NY
      XX15 = X15
  103 CALL IDPLOT
      GO TO 106
  104 CALL IDPLOT
      DO 105 J = 1,52
  105 HEAD(J) = TITLE(J)
      SCALEX =SCX
      SCALEY =SCY
      XB = XXB
      XE = XXE
      YB = YYB
      YE = YYE
      NX = NXX
      NY = NYY
      X15= XX15
  106 CONTINUE
      IF(SCALEX.LT.0.0) SCALEX =  SCALEX
      IF(SCALEX.GT.100.0) RETURN
      IF(SCALEY.LT.0.0) SCALEY = -SCALEY
      IF(SCALEY.GT.100.0) RETURN
      IF(X15.LT.0.0) X15 = -X15
      IF(X15.GT.100.0) RETURN
      X = 0.0
      Y = 0.0
      AN = N
      CALL PLOT( 4.0,1.45,-3)
   12 CALL PLOT(0.0 ,9.0, 2)
      CALL PLOT(X15 ,9.0, 2)
      CALL PLOT(X15 ,0.0, 2)
      CALL PLOT(0.0, 0.0, 2)
      DX=(XE -XB)/SCALEX
```

```
      DY=( YE - YB)/SCALEY
      AX = X15/SCALEX
      AX10 =AX/10.0
      AY = 9.0/SCALEY
      AY10 =AY/10.0
    1 Y=Y+AY10
      IF(Y.GE. 9.0  ) GO TO 2
      CALL PLOT(0.0,Y,2)
      CALL PLOT(0.1,Y,2)
      CALL PLOT(0.0,Y,3)
      GO TO 1
    2 Y = 9.0
      CALL PLOT(0.0,9.0,2)
    3 X = X+AX10
      IF(X.GE.X15 ) GO  TO 4
      CALL PLOT(X ,9.0,2)
      CALL PLOT(X ,8.9,2)
      CALL PLOT(X ,9.0,3)
      GO  TO  3
    4 CALL PLOT(X15 ,9.0,2)
    5 Y =Y -AY10
      IF(Y.LE.0.0) GO TO 6
      CALL PLOT(X15, Y,2 )
      CALL PLOT(X15-.1,Y,2 )
      CALL PLOT(X15 ,Y,3 )
      GO TO 5
    6 X =X15
      CALL PLOT(X15 ,0.0,2)
    7 X = X -AX10
      IF(X.LE.0.0) GO TO 8
      CALL PLOT (X,0.0,2)
      CALL PLOT (X,0.1,2)
      CALL PLOT (X,0.0,3)
      GO  TO  7
    8 X = 0.0
      Y = 0.0
      XS = (X15-13.6)/2.0
      CALL PLOT(0.0, 0.0, 2)
      NNX = 0
      NNY = 0
      IF(XE)200,200,201
  199 IF(YE)210,210,211
  200 XA = ABS(XB)
      GO TO 199
  201 XA=XE
      GO TO 199
  210 YA = ABS(YB)
      GO TO 212
  211 YA = YE
  212 CONTINUE
      IF(XA.LT.1000.) NNX = NX - 3
      IF(YA.LT.1000.) NNY = NY - 3
      IF(XA.LT. 100.) NNX = NX - 2
      IF(YA.LT. 100.) NNY = NY - 2
      IF(XA.LT.  10.) NNX = NX - 1
      IF(YA.LT.  10.) NNY = NY - 1
      IF(XA.LT.   1.) NNX = NX
      IF(YA.LT.   1.) NNY = NY
```

```
      IF(NNX.LT.0) NNX = 0
      IF(NNY.LT.0) NNY = 0
      XSN = NX
      YSN = NY
      SPACEX =  .07*XSN
      SPACEY =  .07*YSN
      IF(XE.LT.0.0) SPACEX = SPACEX + .07
      IF(YE .LT.0.0)SPACEY = SPACEY + .07
      XN = XB
      YN = YB
      CALL NUMBER(X-SPACEX,-.25,.15,XN,0.0,NNX)
      CALL NUMBER(-.10,Y-SPACEY,.15,YN,90.0,NNY)
    9 Y = Y +AY
      YN = YN+DY
      IF(Y.GT.9.0) GO TO 10
      CALL NUMBER(-.10,Y-SPACEY,.15,YN,90.0,NNY)
      CALL PLOT(0.0,Y,3)
      CALL PLOT(X15 ,Y,2)
      Y = Y + AY
      YN = YN + DY
      IF(Y.GT.9.0) GO TO 10
      CALL PLOT(X15,Y,3)
      CALL PLOT(0.0,Y,2)
      CALL NUMBER(-.10,Y-SPACEY,.15,YN,90.0,NNY)
      GO TO  9
   10 X = X +AX
      XN = XN+DX
      IF(X.GT.X15 ) GO TO 11
      CALL NUMBER(X-SPACEX,-.25,.15,XN,0.0,NNX)
      CALL PLOT(X,0.0,3)
      CALL PLOT(X,9.0,2)
      X = X + AX
      XN = XN + DX
      IF(X.GT.X15 ) GO TO 11
      CALL PLOT(X,9.0,3)
      CALL PLOT(X,0.0,2)
      CALL NUMBER(X-SPACEX,-.25,.15,XN,0.0,NNX)
      GO TO 10
   11 CALL SYMBL4(1.65+XS,-.55,.15,HEAD(1),0.0,78)
      CALL SYMBL4(XS, -1.00,.20 ,HEAD(27),0.0,78)
      CALL SYMBL4(1.65+XS,-1.30,.15,HEAD(40),0.0,78)
      CALL SYMBL4(-.35,-.55,.15 ,HEAD(14),90.0,78)
      RETURN
      END
```

## MAIN PROGRAM FOR CALCULATION OF TEMPERATURE FROM SERIES AND PARALLEL RESONANCE FREQUENCIES

```
$IBFTC TEP601
      DIMENSION FMT(14)
   15 FORMAT(12X,2F8.2,F10.2,2F9.0,10X,I6)
      COMMON /KEN/ R, AL, PERCNT, LIMIT, CC, S, TSTART,F,FP,T,KK,N
    1 FORMAT (3F10.2, I5, E15.7, 2F10.2)
      READ 1, R,AL,PERCNT,LIMIT,CC,S,TSTART
      PRINT1, R,AL,PERCNT,LIMIT,CC,S,TSTART
      K=1
      READ 2, FMT
      PRINT2, FMT
    2 FORMAT(13A6, A2)
      CALL CRDIDS
      PRINT 3
    3 FORMAT (1H1, 8X, 4HTIME, 2X, 6HSERIES, 5X, 3HPAR, 5X, 3HSEC, 2X,
     1 8HNEW TEMP, 2X, 8HOLD TEMP, 5X, 3HALT)
    8 READ FMT, F, FP, SEC, OT
      CALL TEMPSP
      PRINT 15, F,FP,SEC,T,OT,K
      PUNCH 15, F,FP,SEC,T,OT,K
      K= K+1
      GO TO 8
      CALL CRD1OF
      CALL EXIT
      END
```

## SUBROUTINE FOR CALCULATION OF THE ETA FUNCTION

```
$IBFTC ETAA
      SUBROUTINE ETAA( Y , ETA, KL)
      REAL K
      N=0
      AJ =0.0
      AL =0.0
      KL= 0
      IF (Y.GT.0.0)GO TO 1
      ETA =1.0E+37
      RETURN
    1 X=1.0
      DX = .5
      DYH = Y +.00001
      DYL = Y -.00001
    2 IF(X.LE.0.0)X =1.0E-37
      KL= KL+1
      IF(X .GT. 1.0E+30) GO TO 5
      IF (N.GT.500) GO TO 5
      PHI = K(0,X)/(X*K(1,X))
      IF(PHI.GE.DYH) GO TO 3
      IF(PHI.LE.DYL) GO TO 4
      ETA = X
      RETURN
    3 AL  = AL +1.0
      AJ  = 0.0
      DX  = DX *AL/2.0
      X = X+DX
      GO TO 2
    4 AJ  = AJ +1.0
      AL  = 0.0
      DX  = DX*AJ/2.0
      X = X -DX
      GO TO 2
    5 PRINT 6 , X ,Y ,DX ,PHI
    6 FORMAT(16H ETA HAS FAILED ,4E20.8)
      ETA = X
      RETURN
      END
```

## SUBROUTINE FOR CALCULATING ELECTRON TEMPEATURE FROM SERIES AND PARALLEL RESONANCE FREQUENCIES

```
$IBFTC TEMPSP
      SUBROUTINE TEMPSP
      COMMON /KEN/ R, AL, PERCNT, LIMIT, CC, S, TSTART, F, FP, T, KK, N
      DATA PI/3.14159265/, EM/9.1083E-31/, EK/1.38044E-23/, G/3.0/,
     1 A/0.0/, FH/1.4/, E00/8.85434E-12/
      PI2= PI*PI
      TCON=(EM/(G*EK))*4.0E+12*PI*PI
      DBYCON=(EK/(4.0* PI2*EM))
      DBYCON = SQRT(DBYCON)*1.0E-06
      CKON= 0.0
      IF(CC .GE. 1.0) GO TO 22
      CKON=(2.0*PI*AL/CC)*E00
   22 FH2= FH*FH
      N=0
      KK=0
      FP2 = FP*FP
      F2 = F*F
      FN2= FP2-FH2
      FC2 = FP2-F2
      IF (FN2 .LE. 0.0) GO TO 28
      IF( F .LT. FH) GO TO 28
      FN= SQRT(FN2)
      SI2= (F2-FH2)/FN2
      S2= 1.0-SI2
      T= TSTART
   32  TEST = T
      DLENGT= DBYCON*SQRT(T)/FN
      RS= R+DLENGT*S
      PHI= SI2*ALOG(AL/RS) - S2*(ALOG(RS/R) + CKON)
      IF (PHI .LE. 0.0) GO TO 28
      CALL ETAA(PHI, ETAS, KL)
      KK=KK+KL
      T=      TCON*RS*RS*FC2              /(ETAS*ETAS)
      N=N+1
      IF (N .GT. LIMIT) GO TO 36
      IF (S .EQ. 0.0) RETURN
      IF ((ABS(T-TEST)/T) .GT. PERCNT) GO TO 32
   28 T= 0.0
      RETURN
   36 PRINT 37, F, FP, T, N, KK
   37 FORMAT (1H1, 21HTOO MANY ITERATIONS (, 2F5.2, F10.0, 2I7, 1H))
      RETURN
      END
```

## MAIN PROGRAM FOR PLOTTING ANTENNA RESISTANCE VERSUS REACTANCE

```
$IBFTC IMPPLT    MAIN PROGRAM FOR PLOTTING ANT Z VS R AND X
C  PROGRAM FOR PLOTTING ANTENNA IMPEDANCE AS A FUNCTION OF R AND X
      DIMENSION HEAD(50),TEN  (11)
      COMPLEX Z,ZO
      LOGICAL  FIRST
      COMMON/IMPCON/Z,F,FP,FNU,FH,ANGLE,T,R,AL
      COMMON /CONSTA/PI,PI2,EMASS,AMASS,Q,CK,EO,G, UO,ALPHAK
      CALL CONSTS
      ALPH=ALPHAK
      READ 1 ,NPLOTS,NRESLV,NTAPE,NSKIP,NPRINT,NPLT,NTHY
      PRINT1 ,NPLOTS,NRESLV,NTAPE,NSKIP,NPRINT,NPLT,NTHY
    1 FORMAT(7I10)
      DO 32 JNP = 1,NPLOTS
      READ 10 ,NDO,KGRID,FMAX,FX
      PRINT 3 ,NDO,KGRID,FMAX,FX
    3 FORMAT(1H1,2I10,2F10.3)
   10 FORMAT(2I10,2F10.3)
      FX=FX*.1E+07
      FMAX=FMAX*.1E+07
      READ 14,FH,FNU,T,R,AL,ANGLE
      PRINT 2,FH,FNU,T,R,AL,ANGLE
    2 FORMAT(1H1,6E13.6)
   14 FORMAT(6E13.6)
   20 FORMAT(2E20.8,F10.3)
   21 FORMAT(1H1,2E20.8)
      READ 20,FNUCON , DEN,ALT
      PRINT 21 , FNUCON , DEN
      CALL GRID(6HVREADR,SX,SY,XB,XE,YB,YE,HX,HY,X15)
      SCX = X15/(XE-XB)
      SCY = 9.0/(YE-YB)
C     PLOT  FOR   10  CONSTANT TEMPERATURES
      DATA TEM/250.,350.,500.,700.,1000.,1400.,2000.,2800.,4000.,5600.
     1 , 4000.
      F = FX
      FP = 0.0
      T = 0.0
      CALL ANTZ
      ZO = Z
      RES =NRESLV
      REF =FMAX/RES
      PRINT 1000
 1000 FORMAT(1H1)
      PRINT70,SCX ,SCY ,F,FP,T,Z,RES,REF
   70 FORMAT(9E14.6)
      PRINT 1000
      DO30JT =1,11
   89 FORMAT(127H      PLOT X        PLOT Y        DP FREQ       PLA FRE
     1Q   TEMPERATURE    RESISTANCE    REACTANCE           FUN
     2 REF   //,19H TEMPERATURE EQUALS,F20.1,8H DEGREES,,12HALT. EQUALS,
     3 F10.2,  3H KM///)
      PRINT 88
   88 FORMAT (1H1)
      PRINT 89,T,ALT
      IF(NTHY.EQ.1) FNU = FNUCON*DEN*T
      IF(FNU.GT.2.*PI*FX) ALPHAK=ALPH*5.0/9.0
```

```
      PXO = 0.0
      PYO = 0.0
      FIRST =.TRUE.
      JL = 0
      DO 28 JF=1,NRESLV
      FJ = JF-1
      FP = FJ*REF
      CALL ANTZ
      Z = Z - ZO
      PXO = PX
      PYO = PY
      PY  =(REAL(Z)-YB)*SCY
      PX  =(AIMAG(Z)-XB)*SCX
      IF(NPRINT.EQ.0.AND.JF/5*5.EQ.JF) PRINT 70,PX,PY,F,FP,T,Z,FNU,REF
      IF(PX.GT.X15) PX = X15
      IF(PX.LT.0.0) PX = 0.0
      IF(PY.LT.0.0)PY=0.0
      IF(P).GT.9.0)PY=9.0
      IF(FIRST)CALL PLOT(PX,PY,3)
      FIRST =.FALSE.
      CALL PLOT(PX,PY,2)
      JL = JL +1
      IF(JL.LE. MPLT) GO TO 28
      JL = 0
      DXX = PX-PXO
      DYY = PY-PYO
  100 IF(DXX) 103,101,103
  101 IF(DYY) 102,28,102
  102 DXX = .1E-06
  103 S = DYY/DXX
      S2 = S*S
      DY =.05*SQRT(S2/(S2+1.0))
      DX =.05*SQRT(1.0/(S2  +1.0))
      IF(S) 104,105,105
  104 DX = -DX
  105 CALL PLOT(PX-DY,PY+DX,3)
      CALL PLOT ( PX+DY,PY-DX,2)
      CALL PLOT( PX, PY, 3)
   28 CONTINUE
   29 T =TEM(JT)
      IF(ALPHAK .NE.ALPH) ALPHAK=ALPH
   30 CONTINUE
   32 CALL PLOT(X15+4.0,-1.45,-3)
      CALL FINI
      STOP
      END
```

Unclassified
Security Classification

**DOCUMENT CONTROL DATA - R&D**

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

| | |
|---|---|
| 1. ORIGINATING ACTIVITY (Corporate author)<br>Upper Air Research Laboratory<br>University of Utah<br>Salt Lake City, Utah | 2a. REPORT SECURITY CLASSIFICATION<br>Unclassified<br>2b. GROUP |

3. REPORT TITLE

Antenna Impedance in the Ionosphere

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Report No. 3

5. AUTHOR(S) (Last name, first name, initial)

Despain, Alvin M.

| 6. REPORT DATE | 7a. TOTAL NO. OF PAGES | 7b. NO. OF REFS |
|---|---|---|
| 13 May 1966 | 145 pages | 43 references |

| | |
|---|---|
| 8a. CONTRACT OR GRANT NO. AF 19(628)-4995* | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>UU 66-7 |
| b. PROJECT NO. 7663 766303 | |
| c. 62405394, 76008015 | 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)<br>AFCRL 66-412 |
| d. 681000, 68920G6395 | |

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

| 11. SUPPLEMENTARY NOTES | 12. SPONSORING MILITARY ACTIVITY |
|---|---|
| *Sponsored in part, by DASA, Wash. D.C. under Web No. 07.007, Project 5710. | Hq. AFCRL, OAR (CRU)<br>United States Air Force<br>L.G. Hanscom Field, Bedford, Mass. |

13. ABSTRACT

The impedance of an antenna probe surrounded by ionospheric plasma is examined. A review of several physical plasma parameters, research techniques, some past experimental results, and various theories is presented. A need is shown for further theoretical sophistication. The theory develops the impedance of an electrically short, cylindrical antenna probe immersed in a warm, lossy, compressible, magneto-ionic, electron fluid. The results are examined and compared to previous work and are applied to several ionospheric experiments. Experimental results as interpreted by the theory are examined and the usefulness of the theory is thus demonstrated. In particular, the theory is shown to be of value in determining electron density, temperature, and collision frequency from RF probe measurements.

Author

DD FORM 1 JAN 64 1473

Unclassified
Security Classification

| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Ionospheric research | | | | | | |
| Antenna impedance probe | | | | | | |
| Ionospheric electron density | | | | | | |
| Ionospheric electron temperature | | | | | | |
| RF probe measurements | | | | | | |
| Parameters of the ionosphere | | | | | | |